Fall 2019

The Evolution of the AP Calculus AB Test: 1955-2018

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The Evolution of the AP Calculus AB Test: 1955 – 2018

By

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B. S., Indiana State University, 2003

Thesis

Submitted in partial fulfillment of the requirements

For the Degree of Master of Science,
With a Major in Mathematics

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2019

TABLE OF CONTENTS

LIST OF TABLES

LIST OF APPENDECIES

LIST OF FIGURES

Abstract

The purpose of this research is to identify and describe the changes to the AP Calculus AB test from its inception in 1955 to the present. Two research questions guided the study. How has the AP Calculus test changed since 1955? How has the student population changed since the AP Calculus AB test began? The AP Calculus AB Test changed along with the Calculus Reform movement to expect to students to have a conceptual understanding. The changes in the presentation of problems on the test and the level of knowledge needed to solve them shows that. The student population has changed in the past twenty years and has increased the number of students of color and female students taking the test.

Keywords: Calculus Reform, AP Calculus

The Evolution of the AP Calculus AB Test: 1955 – 2018

How has Advanced Placement Calculus changed since it was first discussed in 1952? Did the calculus reform of the 1980's affect how AP Calculus is taught and change the curriculum of the course? This paper will continue to look beyond curriculum and how the test has changed to what type of questions are included on the test now. This study will also examine how the availability of technology has allowed the test to evolve from an almost purely algebraic test to a test that expects students to truly have a conceptual understanding of the mathematics. With the availability of data restricted to 1997 and after, changes in race, ethnicity, gender, and location will be studied. This paper will look at the level of mathematics that is needed to find the solution. The number of students taking the test has consistently increased since 1955 to the present with one exception. The first year of the test there were 285 students taking the Mathematics test. There were not any other math tests offered that first year. In 2018, there were 308,538 students taking the AP Calculus AB Test. If all math test takers are included the number is significantly higher. The number rises to 670,415 when the Calculus BC and Statistics tests are included. The Calculus BC test did not exist until 1969 (Bressoud, 2010) and the Statistics test was first offered in 1997. (AMSTATNEWS, 2017)

History of Advanced Placement

Advanced Placement tests and test taking has changed since 1956. In the year prior, there was a push for advanced course work in high school. With the start of the Korean War and the Cold War, there was fear that America did not have the educated adults to be the next set of scientists and engineers that were going to be required for scientific development and national security. According to Rothschild, "Top professionals increasingly needed graduate

work and graduate schools needed strong college graduates. If our high schools were not producing students of talent, America might rot at the core. And if our best high schools and colleges were teaching overlapping material that would be better taught quickly, then somehow we had to speed up the process." (Rothschild, 1999, p. 176) This led to a fund being created by the Ford Foundation named the Fund for the Advancement of Education (FAE). The first program instituted with this money encouraged advanced sophomores in high school to study at the University of Chicago, Columbia University, University of Wisconsin, or Yale University allowing them to complete two years of college-level course work before being eligible for the draft. This did not sit well with the principals of those schools; losing their most advanced students was a hard pill to swallow. After letters were written to FAE, this practice was discontinued, and a different program was found. At Kenyon College a group of faculty was interested in how to get students to pursue an education appropriate to their ability. The problem seemed to be that many high achieving students were going to college and not being challenged. This led to the study of the College Admission with Advanced Standing (CAAS) funded by the FAE. (Rothschild, 1999) There were 11 subject committees including mathematics. The Advanced Placement program was born out of the CAAS and a resulting study was published in 1952 named *General Education in School and College: A Committee Report by the Members of the Faculties of Andover, Exeter, Lawrenceville, Harvard, Princeton and Yale.* (Rothschild, 1999)

In the spring of 1954, the first mathematics examination as well as other subjects was pilot tested with freshmen at 12 colleges and 120 high school students in the 18 participating schools. (Bressoud, 2010) Sometime in 1954 to 1955 the College Board, then the College

Entrance Examination Board, agreed to be the permanent home and to rename the program Advanced Placement. The College Board administered its first Advanced Placement test in the spring of 1956.

The Advanced Placement mathematics test was not strictly calculus when it originated. Topics from Pre-Calculus were taught along with limits, derivatives and integrals. These topics involved all single variable mathematics. The topics were referred to as Math 0, Math 1, and Math 2. These courses covered Pre-Calculus, Calculus 1, and Calculus 2 respectively.

In 1965, the Mathematical Association of America's (MAA) Committee on the Undergraduate Program in Mathematics (CUPM) presented its findings on curriculum. The CUPM stated that Math 0 should be elementary functions and coordinate geometry, or as stated in the paper, Pre-Calculus. (Bressoud, 2010) Math 1 should consist of differential and integral calculus of elementary functions. Math 2 could either be elementary multivariate calculus with Math 3 strengthening those topics, or Math 2 could focus on single variable calculus with Math 3 being exclusively multivariate calculus. When what is now the College Board redesigned AP Calculus, it adopted a majority of the CUPM model. (Bressoud, 2010)

Originally, AP Calculus covered a full year of single variable calculus. This was too intensive for some students to take in high school. Also recognizing that some students did not need all the topics covered to obtain a good foundation, the curriculum was trimmed. This included Math 0 and Math 1 comprising the first semester and Math 1 and Math 2 covering the second semester. To avoid confusion with the AP scores of 1 to 5, the courses were changed to A, B or C. Hence, Calculus AB covers Calculus 1 topics with a review of Pre-Calculus and Calculus

BC covers all of Calculus 1 and Calculus 2. This new program was launched in the fall of 1968 with the first Advanced Placement tests being administered in the spring of 1969.

Next, this paper will examine the attributes of the AP Calculus AB Exam test takers and how they have changed since 1999. These include race, gender and grade level. The college board has been trying to increase the number of diverse students taking AP courses as well as the number of students testing in those courses. Technology will also be discussed. There have been significant changes in the way technology has been used on the test. In turn, technology has changed the way the test is written and administered.

Advanced Placement Calculus AB Test

The Advanced Placement Calculus AB test has changed drastically in some areas and has not changed at all in others. In the beginning the test was called the AP Math test. At some point it was changed to the AP Calculus test but still contained non-calculus questions like trigonometry and Pre-Calculus questions. This changed as time progressed and these topics were eliminated. The test now consists of all calculus and requires multiple processes to solve a problem. The test now also requires the use of a calculator.

Test Questions

The test has changed as the College board has decided to change the requirements of the test since 1957. First semester calculus at the college level has three basic topics, those are limits, differentiation, and accumulations (integral calculus). While these topics have always been present in the AP Calculus AB test there were other topics included. These topics could be considered Pre-Calculus since they are used in the course. However, some of these topics, like logarithms, symmetry, and trigonometry, while they can make calculus easier to understand are

not essential to understanding these three main topics. One of the, most obvious is symmetry. Understanding symmetry can make evaluating functions much easier. For example, a function with origin symmetry, when it is integrated from -a to a, would have a value of 0. Understanding this would make a problem involving this much easier but it is not a necessity in order to understand what an accumulation is.

In 1967, this test had a total of 45 multiple choice questions. Of those 45 questions, nine questions did not require calculus to complete. The decision of whether a problem needs calculus to be completed has been determined by an examination of the solutions that the College Board has provided. The first question (Figure 1) was specifically about symmetry

1. Which of the following defines a function f for which $f(-x) = -f(x)$?

(A) $f(x) = x^2$ (B) $f(x) = \sin x$ (C) $f(x) = \cos x$

(D) $f(x) = \log x$ (E) $f(x) = e^x$

Figure: 1 1969 Calculus AB Test

While comparing this question to the first question from the 1998 test there is a distinct difference in content. (Figure 2).

1. $\int_1^2 (4x^3 - 6x)\,dx =$

(A) 2
(B) 4
(C) 6
(D) 36
(E) 42

Figure 2: 1998 Calculus AB Test

In Figure 1, the student is expected to recognize what an even or odd function is. This is something that could be memorized since the function themselves are common parent functions that could have just been simply memorized. This type of question requires very little analytical thought. In Figure 2, the problem is more than just simple memorization. This question requires the student to apply their knowledge of definite integration. In fact, the test from 1998 is all calculus based.

In the 1973 test, there are only six questions that do not require any calculus to complete. However, these questions require that there is more application of composition and it also requires mathematical reasoning as the question is more than likely one a student has never seen. For example, the problem in Figure 3 requires the student to apply the composition of two functions with one of the functions being a constant.

2. If $f(x) = x^3 + 3x^2 + 4x + 5$ and $g(x) = 5$, then $g(f(x)) =$

(A) $5x^2 + 15x + 25$ (B) $5x^3 + 15x^2 + 20x + 25$ (C) 1125

(D) 225 (E) 5

Figure 3 1973 Calculus AB Test

Another question in the 1973 test is a difficult problem for most students. Again, no calculus is required for the question shown below in Figure 4. To correctly answer this question requires that the student have a good understanding of function notation and be able to solve a system of equations.

12. If $f(x) = 2x^3 + Ax^2 + Bx - 5$ and if $f(2) = 3$ and $f(-2) = -37$, what is the value of $A + B$?

(A) -6 (B) -3 (C) -1 (D) 2

(E) It cannot be determined from the information given.

Figure 4 1973 Calculus AB Test

While the difficulty level of this question requires a higher thought process, it is still not a calculus question. This content is something that is currently taught in most Algebra 2 curriculum. The next question (Figure 5) from the 1973 test is a very complex question that requires students to understand the difference between acceleration and velocity to set the problem up and then requires algebra to solve the problem. The representation of the function changes from a single variable α to function notation and students need to be able to make this distinction to see what the question is truly asking.

13. The acceleration α of a body moving in a straight line is given in terms of time t by $\alpha = 8 - 6t$. If the velocity of the body is 25 at $t = 1$ and if $s(t)$ is the distance of the body from the origin at time t, what is $s(4) - s(2)$?

(A) 20 (B) 24 (C) 28 (D) 32 (E) 42

Figure 5 1973 Calculus AB Test

In the 1985 test, there are five questions that are not calculus-based. These problems involve an understanding of composition of functions, domain, use of trigonometric functions, and applications of symmetry. The problem in Figure 6 is difficult and this is due to most students' difficulties with function notation.

19. If $f(x_1)+f(x_2)=f(x_1+x_2)$ for all real numbers x_1 and x_2, which of the following could define f?

(A) $f(x)=x+1$ (B) $f(x)=2x$ (C) $f(x)=\frac{1}{x}$ (D) $f(x)=e^x$ (E) $f(x)=x^2$

Figure 6 1985 Calculus AB Test

Most students would not even know how to begin to solve the problem. This problem requires students to think in different ways and to even use some trial and error. Since students were not allowed to use a calculator at this point for the test, some students would rely on taking the choices and substituting values for $x_1$and x_2.

The following problem in Figure 7 from the 1985 test does require calculus to solve. It is however a problem that uses pattern recognition and does not lend itself to a deeper understanding of the mathematics.

Also, another problem just like it, was present on the 1988 test (Figure 8). These problems cover the same topic. It is the definition of a derivative. The difference between the two problems is that the problem illustrated in Figure 7 asks for the derivative at a value, while the problem in Figure 8 asks for the derivative with no value. Both questions are related to the definition of the derivative but from a different perspective is. In Figure 7 the problem is to find the definition, whereas in Figure 8 the problem is to use the definition to find the derivative. This type of problem also shows up on the 2008 test.

25. If $f(x) = e^x$, which of the following is equal to $f'(e)$?

(A) $\lim_{h\to 0} \frac{e^{x+h}}{h}$ (B) $\lim_{h\to 0} \frac{e^{x+h} - e^e}{h}$ (C) $\lim_{h\to 0} \frac{e^{e+h} - e}{h}$

(D) $\lim_{h\to 0} \frac{e^{x+h} - 1}{h}$ (E) $\lim_{h\to 0} \frac{e^{e+h} - e^e}{h}$

Figure 7 1985 Calculus AB Test

29. The $\lim_{h\to 0} \frac{\tan 3(x+h) - \tan 3x}{h}$ is

(A) 0 (B) $3\sec^2(3x)$ (C) $\sec^2(3x)$ (D) $3\cot(3x)$ (E) nonexistent

Figure 8 1988 Calculus AB Test

Students would either remember the pattern from the definition of derivatives or not. This is how the definition of derivative was tested without having the students work out a complete solution of the problem. The method that most teachers would use to assess the definition of a derivative is by making the students work them out longhand. This is one option that allows the question writers to assess this topic without having students work a problem with a written response.

In the 1988 test, there are six questions where no calculus is needed to find a solution. These problems again are at the Pre-Calculus level. The question in Figure 9 is a fundamental domain type question. An understanding of domain is really started in an Algebra 2 course, and then again the topic is taught in Pre-Calculus with more in-depth.

2. What is the domain of the function f given by $f(x)=\frac{\sqrt{x^2-4}}{x-3}$?

(A) $\{x: x \neq 3\}$ (B) $\{x: |x| \leq 2\}$ (C) $\{x: |x| \geq 2\}$

(D) $\{x: |x| \geq 2 \text{ and } x \neq 3\}$ (E) $\{x: x \geq 2 \text{ and } x \neq 3\}$

Figure 9 1988 Calculus AB Test

This is a difficult problem for most students. It requires them to recognize the domain issue caused by the rational function and the fact that a square root is being used. Students are usually drilled about not letting the denominator equal zero, but including the square root as well raises the difficulty level of the problem.

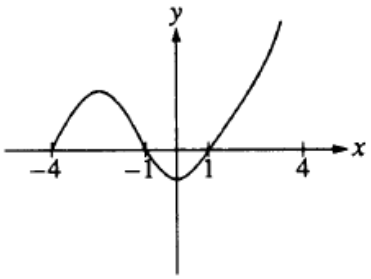
In the 1993 test, there are three problems that do not require calculus to be solved.

13. The fundamental period of $2\cos(3x)$ is

(A) $\frac{2\pi}{3}$ (B) 2π (C) 6π (D) 2 (E) 3

Figure 10 1993 Calculus AB Test

In Figure 10, the question involves rote memorization. The solution guide from the College Board states to use the formula $period = \frac{2\pi}{B}$. Using that formula gives the solution of (A). This topic is taught in most trigonometry courses.



40. The graph of $y = f(x)$ is shown in the figure above. Which of the following could be the graph of $y = f(|x|)$?

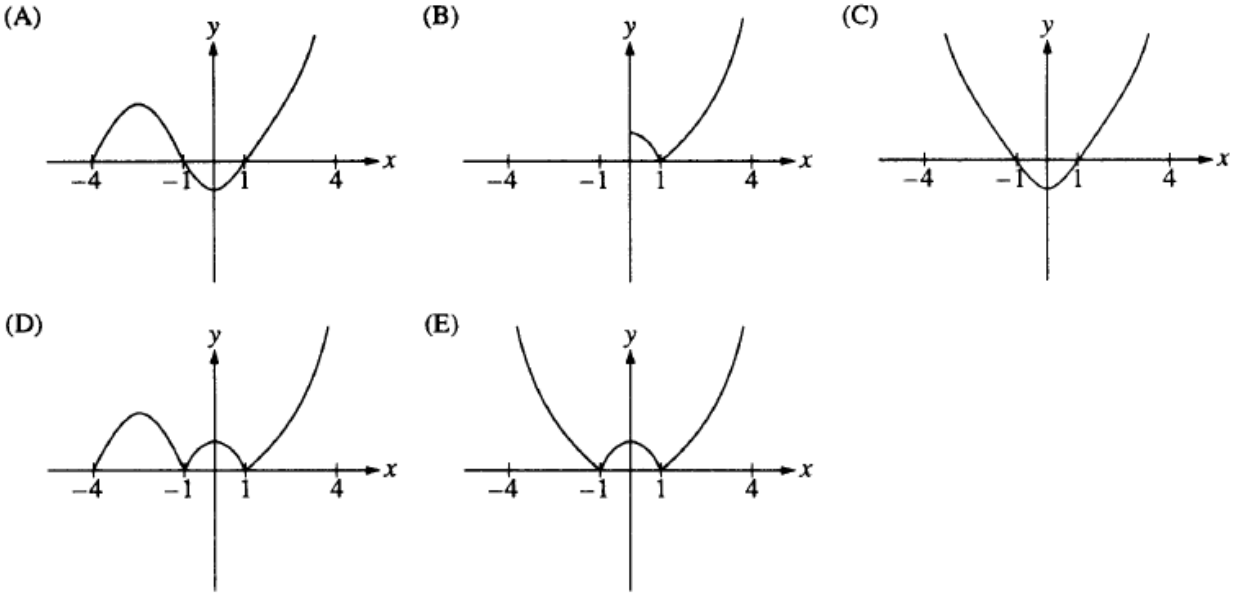
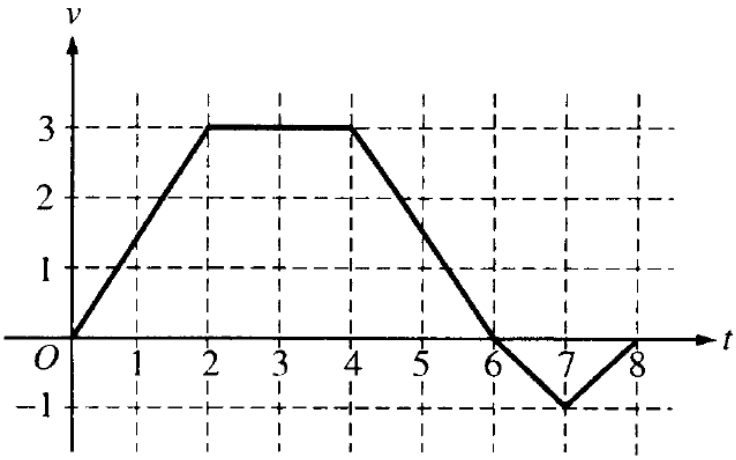


Figure 11 1993 Calculus AB Test

In Figure 11, this is another problem that does not require calculus to solve. This question does require more critical thinking than the question in Figure 10. In Figure 11, this requires a simple idea that the absolute value of x makes the function even. This causes the new function to be symmetrical across the y-axis. That eliminates all solutions except for C and E. Students would then have to recognize that E could not be correct due to the absolute value of x would not affect the x values between 0 and 1. A student does not need any calculus to solve this problem.

In the 1997 test, the year of the redesign, all questions require some knowledge of calculus to be solved. What is meant by that is, for example, in Figure 12 there is an accumulation with a given graph.



9. What is the total distance the bug traveled from $t = 0$ to $t = 8$?

(A) 14 (B) 13 (C) 11 (D) 8 (E) 6

Figure 12 1997 Calculus AB Test

A student could have the knowledge of area under the curve without knowing how to work an integral algebraically. It is also important to note that this if the first year that a problem involving a table is present on the test. In figure 13, this is a trapezoidal approximation. Obviously, area is being found here but with a problem presented in a way that if a student were to know the formula for the area of a trapezoid and be able to apply it four times in problem the test taker would have a chance to get this correct without memorizing the trapezoidal formula that so many students try to memorize and don't understand the simplicity

of it.

x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

89. A table of values for a continuous function f is shown above. If four equal subintervals of $[0,2]$ are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)\,dx$?

(A) 8 (B) 12 (C) 16 (D) 24 (E) 32

Figure 13 1997 Calculus AB Test

Prior to the 1997 test, the questions that were on the test that do not require calculus are good questions that require higher-level thinking. However, because this is an AP Calculus test, the question is, should they be included on the test if they did not require any calculus. Could questions have been written that still required the use of more algebraic skills, but were still calculus level questions? It is apparent and based on what is present in these practice tests, the College Board can write very difficult questions over many topics.

These tests that were given during the years from 1969 to 1997 were extremely algebraic in the sense that questions involved memorization and using specific procedures. The questions rarely asked the test taker to use a graph or a table. In fact, in the 1969 test only one question involved using a graph. In particular, the problem was not presented graphically, rather the solution was a graph. In this test there were no problems posited with a table of information. In fact, as shown in Table 1 the test from 1969 and 1973 contained only 1 graph and no information provided as a table. The first table in the multiple-choice section did not show up until 1997 from released tests. The number of tables showing up on the test increased to four

in 2008. While the number of graphs did increase drastically. In 1997, there were five problems that used a graph in the question or answer. Then by 2008, that number had almost doubled with a total a of nine. The larger increase in the use of tables seem to run concurrently with the allowing calculators and the calculus reform movement.

AP Calculus AB Test		
Year	**Graphs**	**Tables**
1969	1	0
1973	1	0
1985	2	0
1988	2	0
1993	2	0
1997	5	1
1998	8	2
2003	9	2
2008	9	4

Table 1 Number of graphs and tables on AP Calculus AB tests

The use of multiple representations in the form of a graph and/or table was one of the improvements that were a large part of the calculus reform movement. This movement, that will be discussed later in this report, proposed that understanding the concepts of calculus was much more important than working out a problem in a fashion that was algebraically perfect. This reform movement wanted to move away from a focus on procedural knowledge and towards the development of conceptual understanding in calculus. Included in the reform movement was the idea that technology was becoming more accessible for people. When it was first discussed, the cost of graphing calculators was too high to require for a test. The College Board did see the use of calculators as an inevitability in the future.

Use of Technology for Test Taking

Test takers could use calculators for the first time in 1983. This experiment that the College Board and the Educational Testing Service (ETS) tried was discussed first related to the SAT. The ETS statisticians were nervous about changing the way a high-volume test was administered so it was decided by both the ETS and the College Board to try it on a lower volume test. This led to calculators first being used on the 1983 AP Calculus test.

> "It was not technology's finest hour. Conversations among readers at the 1983 AP reading were replete with stories about papers in which insanity would strike good students toward the end of problems causing them to spray ten-digit numbers incoherently across the page." (Kennedy, 2002, p. 576)

The use of calculators was abandoned two years later. An issue here could have been that students were not allowed to use a calculator in their day to day calculus course. Then as students could use them on the test, it would have caused confusion and students also felt that they had to use them and that led to more time being used on questions. (Bressoud, 2019)

From the tests that have been released to the public, a scientific calculator was allowed again in 1993. Students could use the calculator on the entire multiple-choice test. In 1995, after much debate and turmoil between the College Board and graphing calculator companies, graphing calculators were allowed on the AP Calculus tests. The rule for graphing calculators became it could not have a QWERTY style keyboard. That eliminated the calculators that had computer algebra systems. Then within a year or two calculator companies began to release versions that would have this CAS ability without a QWERTY keyboard. Right after that announcement, the College Board announced there would be a non-calculator section to the

test. From the retired tests, this test with the calculator and non-calculator sections seems to be the 1997 test. Since there is no public access to the tests between 1993 and 1997 there is no way to confirm this. However, based on the timeline from several sources this seems to be true. Many may wonder why it took 10 to 11 years for the College Board to adopt these guidelines for the use of technology for test taking. The use of a calculator was not something that the College Board (College Entrance Examination Board) took lightly. The College Board wanted to keep the test equitable for all students. As stated on their webpage, they can make a test for any type of calculator, but they did not want cost or lack of teacher training to be an issue. (College Board, 2004)

Free Response Questions

The free response section of the test is something that has changed right along with the multiple choice section (MC). The amount of free response questions available is quite large however, those sections are only available back to 2000. Since there is nothing available before then, looking at those question in the same way that the multiple-choice is being studied was not feasible for this paper. Two interviews of former readers were conducted. (Tamulis, 2019) (Bressoud, 2019) Both men graded for approximately ten years. The grading process and the training that readers receive was discussed.

According the interview the process for grading has not changed much. The logistics of grading over 400,000 tests is where the struggle really is. The multiple-choice questions require no human grading, the free response questions must be looked at by hundreds of people. The free response questions have changed as the test has changed as well. In every case the FRQ's changed in the same aspects as the multiple-choice questions. As the test was adding tabular

type questions to the multiple-choice sections, there was also the same types of questions being adding the FRQ. The same is true with interpreting graphs. According to Bressoud, between 1993 and 1995 those questions began to show up on the test and students struggled with them as in the first years. As time passed, the students adjusted and interpreting graphs became less of a problem for students. (Bressoud, 2019)

Students Taking the Test

The quantity of students taking the test has increased steadily since 1955.

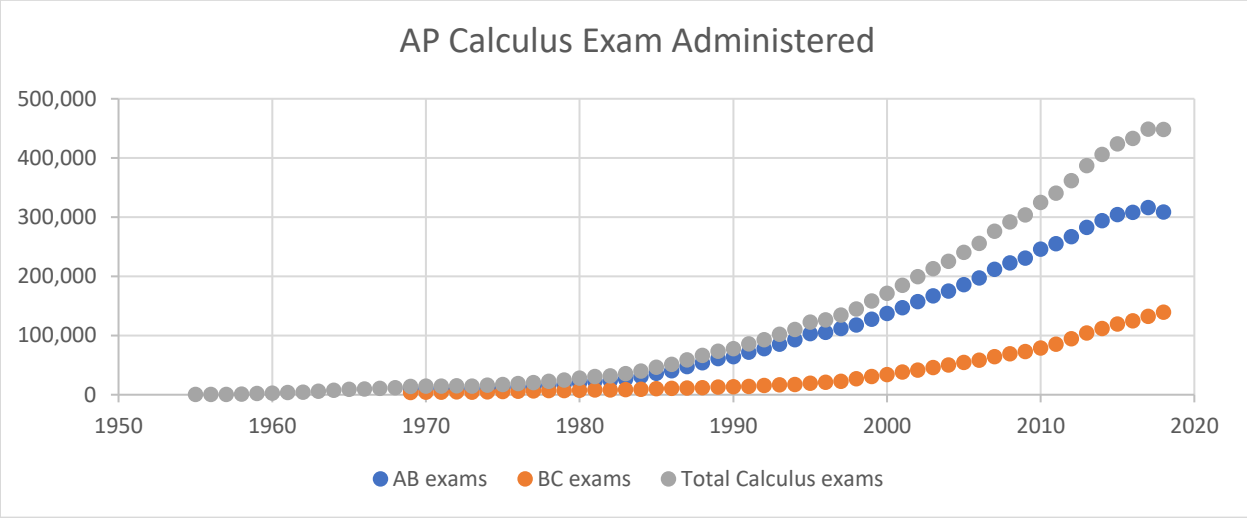


Figure 14 AP Calculus Exams

In 2018, there were 699 fewer students than the previous year. This is not believed to be a statistically significant decrease because it is only a 0.16% decrease for the year. The College Board has made efforts to get more students of color to take the exam. This is evident from the AP Equity and Access Policy from the AP Calculus Course description from the fall of 2019. It states,

> College Board strongly encourages educators to make equitable access a guiding principle for their AP programs by giving all willing and academically prepared students

> the opportunity to participate in AP. We encourage the elimination of barriers that restrict access to AP for students from ethnic, racial, and socioeconomic groups that have been traditionally underrepresented. Schools should make every effort to ensure their AP classes reflect the diversity of their student population. College Board also believes that all students should have access to academically challenging course work before they enroll in AP classes, which can prepare them for AP success. It is only through a commitment to equitable preparation and access that true equity and excellence can be achieved. (College Board, 2019, p. ii)

The diversity of the population of students taking the test has changed in the past 20 years. Since 1997, the data on test takers provided by College Board, has changed drastically. The number of black students taking the exam has increased from 4,019 in 1997 to 16,862 in 2015. The total number students taking the test has also increased in that time, however the percentage of black students testing as a percentage of the total test takers has also increased from 3.7% in 1997 to 5.8% in 2015. Data from 2016 – 2018 is not being discussed because in 2016, the way students were identified was changed. In 2016, there was a decrease in the number of black students because the option of choosing two or more races was added, so it is likely that students who selected two or more races were originally selecting black before 2016. An issue that occurs is that once this new norm was established, the number of black students has been decreasing over the last three years, while the amount of two or more races has been increasing significantly. The only other race to decrease is Native American/Alaskan Native but not near the magnitude of the black students.

The number of white students that took the test has also increased over the same period. However, white students as a percentage of the total number of test takers has decreased from 1997 to 2018. In 1997, 67.5% of test takers were white, then in the 2015 57.4% of test takers were white. Looking at the 2018 data it is even lower but, as stated earlier the way race is identified changed in 2016.

The number of female students taking the test has increased in both numbers and percentages of the total over the same years. In 1997, 51182 females took the test. In 2018, that number had jumped to 148,398. Looking at the percentages over the same time that value went from 47.2% to 49.5%. That means there is only 1% difference in the amount of female and male test takers. This is shown in Figure 15.

The College Board has made great strides to increase the number of females taking the test. The average score for females is lower than the average score for males as shown in Figure 14 the raw data is in Appendix B.

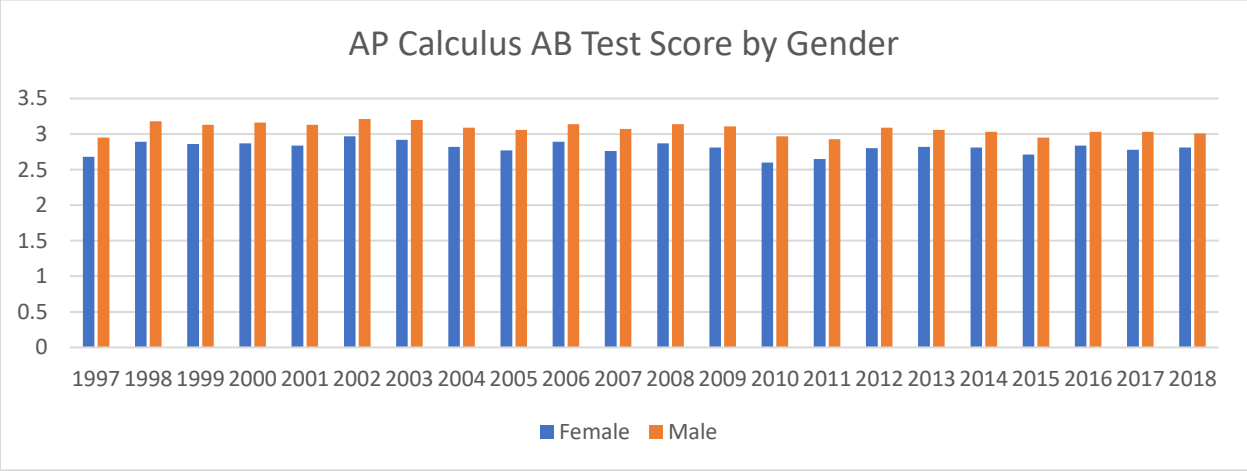


Figure 15 AP Calculus AB Test Score by Gender

This is something that the College Board needs to look at and examine the differences. Something that needs to be studied is how many females are enrolled in AP Calculus and of those, how many are taking the test? This issue may originate from the schools themselves and not the AP tests.

The College Board is constantly trying to reach out to underserved sections of the population whether it be students of color or females. This is evident from the above quote or the article that discusses a partnership with the Dell Foundation in order to get AP Courses into underserved schools. It also states that the biggest issue of getting underserved students enrolled in AP Courses is related to the employees at the school i.e. teachers, counselors etc. This is a problem that is being addressed by the College Board, they have partnered with Khan Academy to give free review of AP and SAT. (College Board, 2014) The College Board also just launch AP Classroom this year to allow all students practice problems that they can work on. This allows the student to work online or for a teacher to have the ability to print practice problems. (College Board, 2019) The College Board is trying to give schools help in teaching AP courses. However, the College Board does not have the ability to go into schools and dictate the way things should be done. Obviously, they do have requirements for being able to offer an AP course.

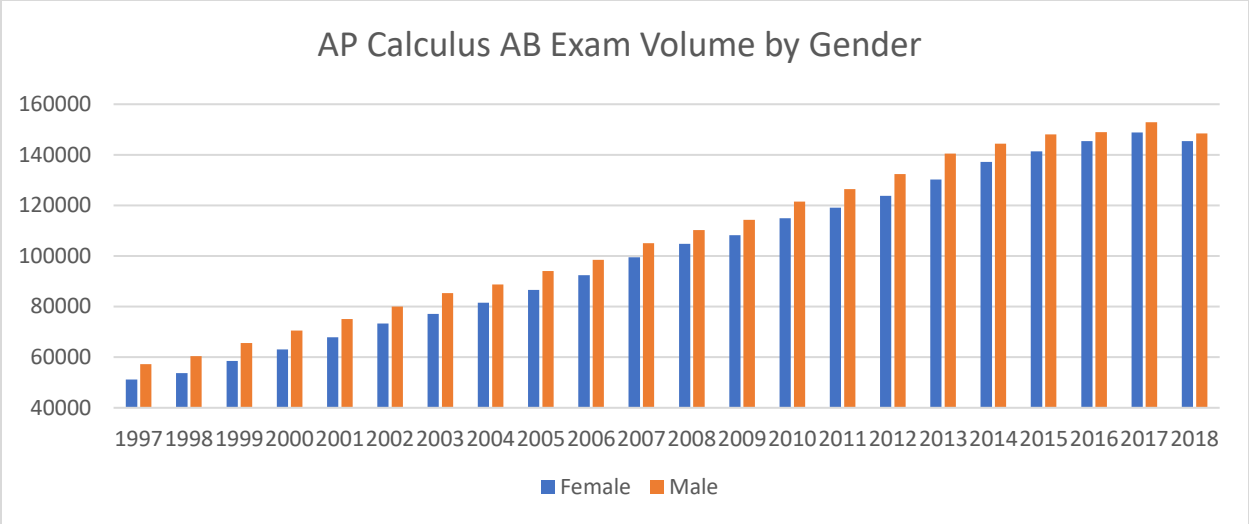


Figure 16 AP Calculus AB Exam Volume by Gender

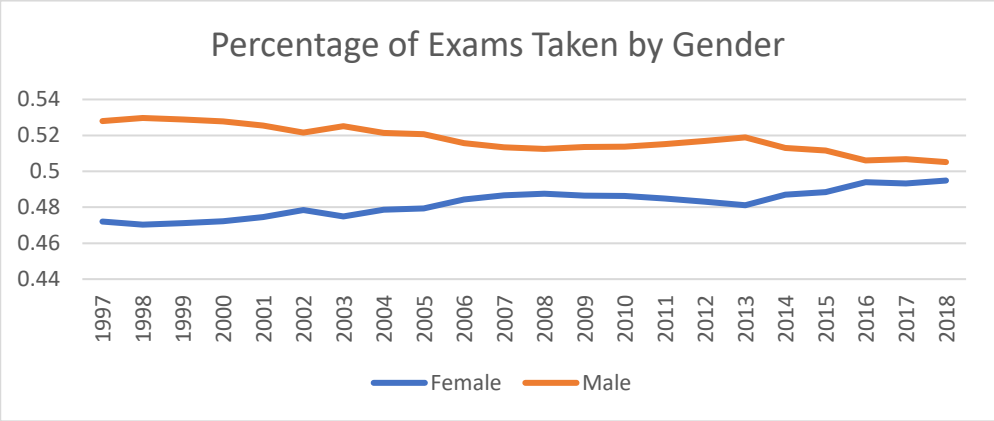


Figure 17 Percentage of Exams Taken by Gender

Calculus Reform

The Calculus Reform movement started officially in January 1986 at the Tulane University Conference. However, according to Meyer (2013), many of the issues that served as a catalyst for the reform had been previously discussed in the mathematics community. The issue with calculus as it was being taught was that the level of conceptual understanding was not at the level it needed to be for students to go beyond it and be able to use it as a tool for other disciplines. There were many factors that the attendees of the Tulane conference believed were

the real reasons for why students were not doing well in calculus. Some did not think that calculus instructors were fully invested in teaching a calculus course. While others believed, instructors were teaching only the mechanics or procedures of solving calculus problems, but not developing the concepts for true understanding. This led to students being able to complete the computations correctly but not understanding the mathematical concepts behind the computations and problem development.

These problems led to a call for a change in both the teaching of and curriculum for calculus. In January of 1986, the conference, chaired by Ronald Douglas and supported by the Sloan Foundation, was held. The paper, *Toward a Lean and Lively Calculus*, published by the Mathematical Association of America (MAA) was the result of the conference. (Haver, 1998) After the conference, funding was needed to move forward with the ideas that were developed. At this time, the National Science Foundation (NSF) did not provide much support for undergraduate education, but this would change a short time later. (Haver, 1998) With the publication of the Neal Report (Haver, 1998) and continued support from the engineering community, the NSF made a formal announcement that it would support change in the calculus curriculum.

Ronald Douglas was the largest supporter of Calculus Reform; in fact, most would say if not for him leading the charge, change would have been difficult. Douglas stated (Peterson, 1987) “We are not doing a good job in teaching what we are teaching. We now have an opportunity to do something about the trouble and to make [calculus] even more important.” On the other side of reform is Leonard Gilman of the University of Texas in Austin stating “There’s no crisis in calculus. We have a solid program and people are learning some

mathematics." (Peterson, 1987, p. 317) Gillman later adds " I have a lot of colleagues who are wedded to their research and don't really care much about calculus [teaching]." (Peterson, 1987, p. 317) He goes on to discuss that there is nothing wrong with sprucing up the curriculum a bit. This apparent change of opinion shows that even a skeptic believed that something should be changed in college-level calculus.

Conclusion

The AP Calculus AB test has changed since the first AP mathematics test was given back in 1956. The level of questioning has changed drastically. The early tests were about algebraic and structured approach to the mathematics. As the Calculus Reform began it caused the College Board to have to change the test accordingly. The College Board needed to be careful and not get ahead of colleges that were accepting the test scores. This led to the test being changed in ways that was more than likely not foreseen in the 1960's. The use of graphing calculators led to students being able to see functions graphed in an instant. Since the tools existed for the test takers to have calculators to do computation, this allowed question writers to expect more from students. That being the case, the emphasis shifted to developing a more conceptual understanding of calculus and not just a procedural approach. Once the Calculus Reform movement encouraged some changes in the test, the College Board then decided to look at the differences in students taking the test. As the years have progressed, the number of students of color, low income students, and female students have increased faster than their counterparts. As the data shows, there is not much to hinder students taking the tests.

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Appendix A
AP Calculus AB Scores by Year and Race

	AP Score	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
AMERICAN INDIAN/ALASKA NATIVE	5	23	32	52	49	39	66	62	65	86	87	84	92	125	101	122	150	167	199	172	87	62	53
	4	55	75	66	80	89	101	87	104	113	120	110	137	143	123	122	139	168	205	172	72	80	69
	3	104	107	112	110	108	129	149	115	115	120	154	146	152	161	157	165	218	206	242	108	129	123
	2	77	86	110	107	99	127	120	130	134	136	143	146	155	100	143	136	163	132	129	87	174	185
	1	104	91	124	149	166	145	210	256	298	293	332	284	300	428	482	505	571	557	576	367	288	266
	T	363	391	464	495	501	568	628	670	746	756	823	805	875	913	1026	1095	1287	1299	1291	721	733	696
MEAN SCORE		2.49	2.67	2.59	2.54	2.47	2.68	2.48	2.39	2.40	2.43	2.36	2.51	2.59	2.31	2.28	2.35	2.38	2.51	2.41	2.20	2.26	2.22
Race total as a percent of total test takers		0.0033	0.0034	0.0037	0.0037	0.0035	0.0037	0.0039	0.0039	0.0040	0.0040	0.0040	0.0037	0.0039	0.0039	0.0042	0.0043	0.0048	0.0046	0.0045	0.0024	0.0024	0.0024
ASIAN	5	2494	3077	3387	3774	3628	4775	5103	5895	6646	7862	8008	8976	10234	9706	11173	13640	14082	15914	14785	16956	13875	14823
	4	3492	4070	4277	4714	5037	5304	5517	5070	5533	6132	6279	7238	6893	6628	7118	7608	9035	8671	9106	9069	10254	9684
	3	4423	4475	4509	4658	5272	5919	5749	4823	4762	5287	6015	5728	6176	6837	7260	7218	7876	8180	9222	8105	10362	10373
	2	2982	2761	3325	3912	3731	3966	3623	4277	4424	4147	4744	4732	5089	4151	4093	4095	4849	4684	4876	4276	9647	9699
	1	2792	2476	2864	3222	3442	3175	3731	5046	5769	5621	6876	6473	6958	10286	10481	9959	10694	11715	12928	11013	7439	6991
	T	16183	16859	18362	20280	21110	23139	23723	25111	27134	29049	31922	33147	35350	37608	40125	42520	46536	49164	50917	49419	51577	51570
MEAN SCORE		2.99	3.15	3.11	3.09	3.08	3.20	3.20	3.10	3.11	3.22	3.12	3.23	3.24	3.04	3.11	3.26	3.24	3.25	3.16	3.34	3.26	3.30
Race total as a percent of total test takers		0.1492	0.1478	0.1479	0.1519	0.1489	0.1509	0.1461	0.1474	0.1467	0.1521	0.1561	0.1541	0.1589	0.1590	0.1634	0.1660	0.1719	0.1746	0.1759	0.1678	0.1710	0.1755
BLACK	5	126	204	228	277	252	326	341	417	472	544	512	666	715	666	767	1095	1235	1357	1159	1405	900	907
	4	372	455	539	613	627	715	783	703	779	906	853	1114	1014	976	1043	1246	1607	1561	1589	1597	1399	1334
	3	776	908	920	996	1042	1259	1323	967	974	1181	1213	1353	1394	1342	1572	1639	1895	2090	2207	1960	2319	2367
	2	832	944	1112	1225	1168	1259	1319	1230	1250	1357	1417	1561	1745	1199	1243	1324	1636	1599	1671	1386	3604	3471
	1	1913	1825	2201	2369	2635	2612	2912	3613	4367	4465	5334	5596	6457	8022	8842	8548	8535	9367	10236	9031	7221	6456
	T	4019	4336	5000	5480	5724	6171	6678	6930	7842	8453	9329	10290	11325	12205	13467	13852	14908	15974	16862	15379	15443	14535
MEAN SCORE		2.00	2.14	2.10	2.12	2.07	2.17	2.15	2.00	1.95	2.02	1.91	2.00	1.92	1.78	1.79	1.92	2.02	1.99	1.92	2.02	2.04	2.09
Race total as a percent of total test takers		0.0371	0.0380	0.0403	0.0410	0.0404	0.0402	0.0411	0.0407	0.0424	0.0443	0.0456	0.0478	0.0509	0.0516	0.0548	0.0541	0.0551	0.0567	0.0582	0.0522	0.0512	0.0495
HISPANIC/LATINO	5	337	442	525	669	525	772	958	1145	2269	1570	1633	1957	2155	2274	2622	3547	4156	4643	4092	6432	4778	4887
	4	705	891	1023	1152	1041	1417	1641	1532	2428	1979	1880	2760	2653	2442	2735	3472	4371	4439	4504	6189	6074	6015
	3	1128	1279	1424	1466	1502	2186	2333	1812	2807	2277	2694	2960	3240	3570	4047	4327	5178	5721	6212	7599	9133	9041
	2	1132	1189	1422	1695	1428	1968	2199	2272	3095	2450	2848	3289	3719	2639	2856	3134	4043	4127	4321	4912	13331	12885
	1	1842	1829	2135	2635	3017	3310	4109	5423	7520	6764	8274	8864	9944	13128	14955	15026	16978	19125	21686	24615	20231	18617
	T	5144	5630	6529	7617	7513	9653	11240	12184	18119	15040	17329	19830	21711	24053	27215	29506	34726	38055	40815	49747	53547	51445
MEAN SCORE		2.33	2.45	2.45	2.41	2.29	2.42	2.39	2.24	2.38	2.28	2.18	2.28	2.23	2.09	2.09	2.23	2.27	2.25	2.14	2.29	2.29	2.33
Race total as a percent of total test takers		0.0474	0.0493	0.0526	0.0570	0.0530	0.0630	0.0692	0.0715	0.0980	0.0788	0.0847	0.0922	0.0976	0.1017	0.1108	0.1152	0.1283	0.1351	0.1410	0.1689	0.1775	0.1751
NATIVE HAWAIIAN/OTH PACF ISL	5																				78	46	42
	4																				66	76	52
	3																				73	90	89
	2																				42	114	102
	1																				192	151	134
	T																				451	477	419
MEAN SCORE																					2.55	2.48	2.44
Race total as a percent of total test takers																					0.0015	0.0016	0.0014

Appendix A (cont.)

	AP Score	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
WHITE	5	8465	12414	13733	16164	16788	19923	21572	24936	26251	28847	29508	32274	33922	32994	33849	41175	41402	42806	38321	42748	32079	30659
	4	15356	18486	19585	22769	24374	25661	27388	24689	25344	27183	27114	31847	28649	26066	26986	28714	31969	30033	31785	31455	33298	30201
	3	20472	21202	21476	22515	25127	28925	28426	23105	22551	24164	26918	26396	26934	28139	30124	28895	30217	31944	33847	31004	37636	36180
	2	14837	13706	15946	18143	17034	17721	17992	20746	20650	19434	20955	21566	22426	16996	16754	16062	18875	18482	17809	16824	36512	36366
	1	14089	10959	12723	13257	16055	13692	16974	23228	26614	24562	29478	27718	29817	42088	43539	39561	40381	42951	44571	42029	24036	24061
	T	73219	76767	83463	92848	99378	105922	112352	116704	121410	124190	133973	139801	141748	146283	151252	154407	162844	166216	166333	164060	163561	157467
MEAN SCORE		2.85	3.10	3.07	3.11	3.09	3.19	3.17	3.06	3.03	3.13	3.05	3.14	3.10	2.94	2.94	3.10	3.09	3.07	3.01	3.10	3.08	3.04
Race total as a percent of total test takers		0.6752	0.6728	0.6723	0.6954	0.7010	0.6908	0.6918	0.6852	0.6563	0.6504	0.6550	0.6500	0.6370	0.6185	0.6159	0.6028	0.6015	0.5901	0.5745	0.5572	0.5422	0.5360
TWO OR MORE RACES	5																				2883	2510	2652
	4																				1993	2375	2339
	3																				1901	2770	2852
	2																				1106	2802	2823
	1																				3512	2395	2307
	T																				11395	12852	12973
MEAN SCORE																					2.97	2.98	3.02
Race total as a percent of total test takers																					0.0387	0.0426	0.0442
OTHER	5	290	455	507	737	602	776	893	1007	1085	1162	1202	1483	1503	1366	1461	1817	2091	2309	2030	111	13	1
	4	490	684	739	904	1005	1016	1042	966	982	1129	1178	1448	1202	1122	1126	1184	1483	1460	1471	75	14	1
	3	721	723	854	1017	1135	1158	1138	986	925	1014	1191	1154	1185	1218	1244	1189	1450	1522	1653	66	8	2
	2	523	545	699	782	720	769	796	901	865	875	1007	1013	1004	715	720	699	922	867	874	45	14	
	1	546	504	642	704	909	742	836	1167	1328	1291	1563	1653	1636	2269	2311	2178	2534	2604	2664	99	17	1
	T	2570	2911	3441	4144	4371	4461	4705	5027	5185	5471	6141	6751	6530	6690	6862	7067	8480	8762	8692	396	66	5
MEAN SCORE		2.79	3.01	2.93	3.05	2.92	3.07	3.08	2.95	2.93	3.00	2.91	3.01	2.99	2.79	2.81	2.97	2.96	3.00	2.92	3.14	2.88	3.20
Race total as a percent of total test takers		0.0237	0.0255	0.0277	0.0310	0.0308	0.0291	0.0290	0.0295	0.0280	0.0287	0.0300	0.0314	0.0293	0.0283	0.0279	0.0276	0.0313	0.0311	0.0300	0.0013	0.0002	0.00002
NO RESPONSE	5	925	1363	1331	498	522	623	545	761	896	1782	962	915	1055	1645	1001	1760	354	436	717	703	682	921
	4	1594	1809	1707	655	795	844	723	767	883	1639	897	926	891	1379	899	1250	290	273	600	461	561	721
	3	1974	1974	1725	631	769	853	735	676	837	1513	983	843	888	1509	1023	1348	280	359	730	423	632	900
	2	1274	1152	1176	483	484	563	501	646	765	1237	749	670	777	981	608	842	219	231	447	272	716	959
	1	1172	911	945	385	593	526	572	854	1164	1824	1438	1108	1384	3236	2094	2516	797	884	2103	1029	841	1166
	T	6939	7209	6884	2652	3163	3409	3076	3704	4545	7995	5029	4462	4995	8750	5625	7716	1940	2183	4597	2888	3432	4667
MEAN SCORE		2.97	3.22	3.19	3.15	3.05	3.14	3.05	2.98	2.91	3.04	2.84	2.97	2.89	2.68	2.66	2.86	2.58	2.61	2.43	2.84	2.86	2.84
Race total as a percent of total test takers		0.0640	0.0632	0.0555	0.0199	0.0223	0.0222	0.0189	0.0217	0.0246	0.0419	0.0246	0.0207	0.0224	0.0370	0.0229	0.0301	0.0072	0.0078	0.0159	0.0098	0.0114	0.0159
NATIONAL TOTAL	5	12660	17987	19763	22168	22356	27261	29474	34226	37705	41854	41909	46363	49709	48752	50995	63184	63487	67664	61276	71403	54945	54945
	4	22064	26470	27936	30887	32968	35058	37181	33831	36062	39088	38311	45470	41445	38736	40029	43613	48923	46642	49227	50977	54131	50416
	3	29598	30668	31020	31393	34955	40429	39853	32484	32971	35556	39168	38580	39969	42776	45427	44781	47114	50022	54113	51239	63079	61927
	2	21657	20383	23790	26347	24664	26373	26550	30202	31183	29636	31863	32977	34915	26781	26417	26292	30707	30122	30127	28950	66914	66490
	1	22458	18595	21634	22721	26817	24202	23344	39587	47060	44820	53295	51696	56496	79457	82704	78293	80490	87203	94764	91887	62619	59999
	T	108437	114103	124143	133516	141760	153323	162402	170330	184981	190954	204546	215086	222534	236502	245572	256163	270721	281653	289507	294456	301688	293777
MEAN SCORE		2.82	3.04	3.00	3.03	3.00	3.10	3.07	2.96	2.93	3.02	2.92	3.01	2.97	2.79	2.80	2.95	2.94	2.92	2.83	2.94	2.91	2.91

Appendix B
Test Takers by Gender

	Average Score		# of Test Takers		% of Total	
Year	**Female**	**Male**	**Female**	**Male**	**Female**	**Male**
1997	2.68	2.95	51182	57255	47.2%	52.8%
1998	2.89	3.18	53665	60438	47.0%	53.0%
1999	2.86	3.13	58487	65656	47.1%	52.9%
2000	2.87	3.16	63037	70479	47.2%	52.8%
2001	2.84	3.13	67836	75108	47.5%	52.5%
2002	2.97	3.21	73351	79972	47.8%	52.2%
2003	2.92	3.2	77127	85275	47.5%	52.5%
2004	2.82	3.09	81521	88809	47.9%	52.1%
2005	2.77	3.06	86611	94050	47.9%	52.1%
2006	2.89	3.14	92469	98485	48.4%	51.6%
2007	2.76	3.07	99538	105008	48.7%	51.3%
2008	2.87	3.14	104843	110243	48.7%	51.3%
2009	2.81	3.11	108249	114285	48.6%	51.4%
2010	2.6	2.97	114989	121513	48.6%	51.4%
2011	2.65	2.93	119088	126484	48.5%	51.5%
2012	2.8	3.09	123767	132396	48.3%	51.7%
2013	2.82	3.06	130268	140453	48.1%	51.9%
2014	2.81	3.03	137186	144467	48.7%	51.3%
2015	2.71	2.95	141395	148112	48.8%	51.2%
2016	2.84	3.03	145463	148993	49.4%	50.6%
2017	2.78	3.03	148811	152877	49.3%	50.7%
2018	2.81	3.01	145379	148398	49.5%	50.5%

Compiled from National AP Report 1997-2018

Appendix C
Exams By Year

Year	AB exams	BC exams	Total Calculus exams
1955			285
1956			380
1957			750
1958			1,177
1959			1,870
1960			2,908
1961			3,609
1962			4,190
1963			5,858
1964			7,710
1965			9,021
1966			9,630
1967			10,703
1968			11,623
1969	10,280	3,674	13,954
1970	10,273	4,106	14,379
1971	10,592	4,214	14,806
1972	10,611	4,575	15,186
1973	9,871	4,439	14,310
1974	11,213	4,825	16,038
1975	11,804	5,286	17,090
1976	13,076	5,989	19,065
1977	14,089	6,228	20,317
1978	15,774	6,736	22,510
1979	17,615	7,112	24,727
1980	20,096	7,783	27,879
1981	22,537	8,021	30,558
1982	23,825	8,093	31,918
1983	26,706	8,783	35,489
1984	30,583	9,379	39,962
1985	36,282	10,069	46,351
1986	40,790	10,717	51,507
1987	47,783	11,340	59,123

Year	AB exams	BC exams	Total Calculus exams
1988	54,235	11,992	66,227
1989	60,869	12,778	73,647
1990	64,371	13,239	77,610
1991	71,769	13,939	85,708
1992	77,557	15,639	93,196
1993	85,351	16,594	101,945
1994	93,077	17,431	110,508
1995	103,032	19,581	122,613
1996	105,402	21,186	126,588
1997	111,834	22,668	134,502
1998	117,671	27,088	144,759
1999	127,744	30,724	158,468
2000	137,276	34,142	171,418
2001	146,771	38,134	184,905
2002	157,524	41,785	199,309
2003	166,821	45,973	212,794
2004	175,094	50,134	225,228
2005	185,992	54,415	240,407
2006	197181	58,603	255,784
2007	211,693	64,311	276,004
2008	222,835	69,103	291,938
2009	230,588	72,965	303,553
2010	245,867	78,998	324,865
2011	255,357	85,194	340551
2012	266,994	94,403	361397
2013	282,814	104,483	387297
2014	294,072	112,113	406185
2015	304,345	119,565	423910
2016	308,215	124,931	433146
2017	316,099	132,514	448613
2018	308,538	139,376	447914

Appendix D
AP Calculus AB Exams Not Requiring Calculus

AP Calculus AB Test	
Year	Questions
1967	9
1973	6
1985	5
1988	6
1993	3
1997	0
1998	0