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Quantitative Urbanism: A Look at Illinois Cities Using Modeling

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By

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Abstract: Quantitative urbanism is described as the field of study that explores the social, economic, and physical principles that cities are a product of. This new field of mathematics is quickly growing as various disciplines are attempting to better understand urban growth. This paper will explore the latest discoveries in the relationships that exist between cities and their size. Many aspects of cities, such as crime rates, energy usage, and wealth, have been shown to change exponentially in relation to city size. This paper explores multiple urban indicators vs. population size for cities in Illinois and discusses the results compared to current research. The paper discusses briefly how this developing data could be used in the future as our world become more and more urban. Possibly, what we know about successful urban regions could help us build strong new communities in developing countries.

<u>Relevant History and Literature:</u>

When discussing the future of cities, many indicators point toward a large increase in urbanization over the next century [1, 2, 3, 4, 5, 6, 7]. According to the *United Nations World Urbanization Prospects* [1], the urban population of the world has grown rapidly since 1950. In 1950, only 30% of the world's population was urban and by 2014, this percentage reached 54%. By 2050, this statistic is predicted to rise to 66% due to continuing population growth and urbanization. This means 2.5 billion more people are expected to live in urban regions over the next couple of decades. The United Nations states that "by 2030, the world is projected to have 41 mega-cities with more than 10 million inhabitants" each [1: 1]. Ninety percent of this increase is expected to occur in Asia and Africa, which are quickly growing. According to the United Nations World Urbanization Prospects [1: 1], "just three countries—India, China, and Nigeria—together are expected to account for 37% of the projected growth of the world's urban population". For example, New Delhi is expected to grow from 25 million current inhabitants (in 2014) to 36 million inhabitants by 2050.

This rush towards urbanization has created concerns in many different disciplines, regarding resources, city design, economics, and the environment. Throughout history, the theories and ideas about the management of cities have evolved tremendously. In *The Kind of Problem a City Is* [8: 3], Luís Bettencourt stated that the discussion of cities and their form goes back at least as far as "the debates between Plato and Aristotle about the nature of human societies". In his work *Politics*, Aristotle referred to people as the most "political" of all animals.

He discussed how city-states arise out of nature, along with idea that the city-state and political rule are “natural”. [9] Evidence of even earlier forms of city planning appear in Greece as far back as the fourth and fifth centuries B.C., where changes were made to incorporate private houses outside of the city center. In western Asia, excavations of Babylonia and Assyria have shown that streets were designed for kings to travel efficiently from temples to palaces. [10] Later, in Roman times, the streets of cities and the land designations within the city limits were determined by professional land surveyors, who had training in Greek geometry [11]. From this time until the first industrial revolution, city planning practices consisted of informal building procedures and architecture driven by time period, region, and culture [8, 12, 13].

In the late 1800s, the management of cities changed as new technology became available and mass production of building materials began. The availability of better transportation and more standardized materials allowed urban planning to expand. Two evolutionary lines of thought on cities emerged at this time, where some viewed cities as systems that can be optimized and others expressed cities as open-ended processes that are subjective to evolution. [8] In 1915, the second of these evolutionary views of cities was mentioned by a biologist, Patrick Geddes [5], who saw cities as an ecological entity and referenced planning as a way to guide evolution, not determine it. This view is carried on today with planners who work to respect historical architecture and advocates of generative urbanism [13, 14].

The view of cities as a system to optimize has generated a tradition that considers cities as a set of problems that need to be managed or redesigned [8]. According to Bettencourt in *The Kind of Problem a City Is,* urban planners and engineers have been influenced by engineering practices and viewed cities as machines that can be improved using control theory [8]. Throughout the 1800s and early half of the 1900s, urban movements focused on optimizing various aspects of cities, including auto transportation routes, green space, disease control, and standardized housing [15, 16]. These concepts, used in isolation, are considered outdated in developed countries, as they need not meet the social needs of the citizens. However, control theory is being used in the developing nations that are growing quickly. The notion that cities are something to control and force change upon is frustrating to many scientists and mathematicians, who believe it is detrimental to try to control a city and constrain its evolution. [8]

This leads to a discussion of the psychology of cities, which presents the latest view of city planning. Recent research shows that human social interactions, organizations, and dynamics create the integrated social networks that form cities [2, 8, 17, 18]. One leader in the field of quantitative urbanism, Geoffrey West [19], described the hierarchical nature of human relationships as the root of cities:

> First of all, you cluster in a family. On average, an individual doesn't have a powerful connection with more than four to six people, and that's just as true here in the U.S. as it is in China. Then there are clusters of families, and then larger clusters that form neighborhoods, and so on, all the way up…They could be the universal thing holding the city together.

These social networks are believed to contribute to the growth, innovation, and wealth of cities by creating opportunities for connections, networking, and generation of new ideas [2, 8, 17, 18]. Researchers from many different fields are exploring the social aspects and other factors of cities to create a greater understanding of urban development.

With the World Bank [20] expecting a 3-fold increase in urban populations in the future, the need to balance urban expansion and its impact on the environment is very urgent [7]. Several studies [21, 22, 23] indicate that cites are complex systems of some sort, but researchers at the Santa Fe Institute and around the world are working to determine if cities can be modeled differently. They explored if cities are most similar to biological systems, (since mathematics has been used previously to scale and quantify biological organisms or ecosystems), or if they are a new type of system [24]. In the article *Growth, Innovation, Scaling, and the Pace of Life in Cities* [18: 7301], researchers from the Santa Fe Institute (Luís Bettencourt, José Lobo, Geoffrey West) and collaborators (Dirk Helbing, Christian Kürnert) described the challenge of the future as trying to "understand and predict how changes in social organization and dynamics resulting from urbanization will impact the interactions between nature and society". If nations cannot understand cities well enough to plan effectively for the future, they may run out of resources (both non-renewable and renewable).

Another problem generated from the recent increases in urbanization is a lack of living structures. UN-HABITAT [6] predicted as many as a billion people world-wide today live in slums and many others build their own shelters without community resources. In Mumbai and New Dehli, 80 to 85% of people who migrate to these cities are absorbed by the slums. India's capital has a slum population over 10 million and as many as 400,000 people per year move into these slums. [25] In *Planet of the Slums* [25], Mike Davis discussed that the cost of new urbanization without proper planning will be increasing inequalities within and between cities of different sizes and economic statuses. The United Nations emphasized that there are very few plans to accommodate these people or provide them with necessary services [6]. A greater understanding of developed large cities could help plan for and navigate the fast growth of developing cities.

These pressing issues have created a need, and desire, to better understand cities and what defines them. Recent technology has created many new opportunities for gathering and analyzing data on large and small cities internationally. The availability of large amounts of data generated by technology has helped develop the new term “big data”, which is an accumulation of data that is too large and complex for processing by traditional database management tools. As stated in *Big Data and City Living-What Can It Do for Us?* [26: 4], instead of small-scale studies and surveys, data is now coming from “mobile technology, embedded sensors, blogs, social media, and location-based tools”. This data boom has allowed researchers to model cities in various ways to try to gain a better understanding of how they work and how they are related.

With this big data, some mathematicians, physicists, sociologists, and others are exploring scaling laws as a means for understanding cities. For over 60 years, biologists have used complex systems and scaling laws to analyze biological design and behavior in animals. When Geoffrey West [19] started his work in scaling laws by exploring metabolism, he recognized that the scaling laws already discovered didn't have accepted explanations. In an interview [19], West discussed his work with the first and most famous biological scaling function, Kleiber's Law, which describes how metabolic rate is related to an organism's size. The law states that metabolic rate (*r*) is the mass (*M*) of an organism raised to the three-quarters power ($r = M^{3/4}$). For example, a whale weighs about 100 million times more than a shrew, but a whale's metabolic rate is only a million times larger. This natural phenomenon means that larger animals consume less energy per unit of time and mass. Surprisingly, it holds true for almost all organisms. [19] In order to confirm Kleiber's Law, West found other scaling laws evident in organisms, including ones relating heart rate, mass, and life span of animals. He applied these findings to his exploration in biological networks and created a model for the mammalian circulatory system. The model represented the connections between blood flow rate, metabolic rate, and mass in mammals. According to Bettencourt, West, and Lobo in *Growth, Innovation, Scaling, and the Pace of Life in Cities* [18: 7302],

> Conceptually, the existence of such universal scaling laws implies, for example, that in terms of almost all biological rates, times, and internal structure, an elephant is approximately a blown-up gorilla, which is itself a blown-up mouse, all scaled in an appropriately nonlinear, predictable way.

Since West's work in the early 2000s [19], scaling laws have been applied to trees and plants. For example, the number of branches on a tree was found to scale to the radius of a tree trunk.

Previous Work in Urban Scaling Laws:

Since cities have been historically compared to biological entities, West started to consider whether cities could be compared to the biological organisms' natural scaling laws [8, 18, 24, 19]. In 2003, at a Santa Fe Institute workshop on modeling aspects of human society, West questioned whether the biological principles from his previous work might apply to human institutions. Luís Bettencourt and José Lobo suggested that they get the data to test the idea and explore if scaling laws apply. [24] "Significant obstacles toward this goal are the immense diversity of human activity and organization and an enormous range of geographic factors" cited Bettencourt, et al [18: 7301]. However, the research team found previous evidence and studies citing increases in economics, innovation, and pace of life between smaller and larger cities [18]. Additionally, Zipf's Law (which shows rank-size distribution) has already been explored using cities.

Research has determined that Zipf's Law applies to population size, income distribution, and number of cities [27, 28, 29, 30, 31, 32, 33, 34]. For example, "the number of cities in individual countries follows an inverse power relationship; the number of cities in the first largest country is twice as many as that in the second largest country, three times as many as that in the third largest country, and so on" [34: 2]. West and Bettencourt [18] gathered data and statistics predominantly from the United States, with some data from Germany and China. Data from a variety of sources including the U.S. Census Bureau, Eurostat Urban Audit, and China's National Bureau of Statistics and contained information on infrastructure, gas stations, income, disease, education, crime, business, and even walking speed. [18, 35] In the U.S., the data was sorted by MSA (Metropolitan Statistical Areas) instead of political or geographic boundaries.

Using this data, Bettencourt, Lobo, Kürhert, and West explored scaling relations for cities looking at “energy consumption, economic activity, demographics, infrastructure, innovation, employment, and patterns of human behavior” [18: 7302]. Twenty indictors were assessed, including new patents, employment, GDP, housing, electrical consumption, gasoline use, disease, and crime. The researchers used ordinary least-squares methods to search for types of functions that fit the data.

In the work of Bettencourt, et.al [18], regression analysis showed that the data were best fit with power law functions. The power law scaling developed from the analysis is $Y(t) = Y_0 N(t)^{\beta}$, where $N(t)$ is the measure of city size at time *t*. *Y* represents the indicator being measured and Y_0 is a constant that normalizes the scaling. The exponent, β, shows the scaling factor for the particular urban indictor being discussed. The researchers found that each indicator had a β value between .77 and 1.34. A scaling exponent of less than one means that the urban indicator has decreasing returns with increases in city size, while an exponent of one represents a change in indicator that is linear as the city size increases. A β value greater than one demonstrates increasing returns on the indicator as city size increases.

The results show that many properties of cities are universal. They appear to be independent of population, location, and time, and truly are scaled versions of one another. The researchers found that linear relationships ($\beta \approx 1$) are associated with citizens' needs (ie. jobs, housing), while sublinear relationships ($\beta < 1$) represent changes in infrastructure (ie. roads, gasoline stations). Superlinear relationships ($\beta > 1$) are of special interest, as they relate to the social aspects of cities. The models showed that patents, inventions, employment, wages, GDP, and crime increased superlinearly with city size. This suggests that perhaps the social indicators of cities are strongly influenced by the size of the city.

Concerns with Heteroskedasticity:

Since very large sets of data were used, the data were tested and adjusted for heteroskedasticity. Heteroskedasticity (Fig. 1) refers to when the variability of a variable is unequal across the range of the second variable that predicts it. This will often create a cone-shaped scatterplot, where the variability (and therefore the residuals) of the data increases along the axis of the input. Homoskedasticity (Fig. 2) is the absence of heteroskedasticity. Homoskedastic describes a graph where the residuals are relatively constant and do not depend on the independent or predictor variable.

For example, yearly vacation expenditures might be a heteroskedastic variable when predicted by income. Families with low incomes will likely have small vacation expenditures, while as families have increasing incomes, their vacation expenditures will have a lot of variability. Some high income families will choose to have minimal vacation expenditures, while others will spend a great amount of their income on vacations, so the gap between the vacation expenditures will increase as income increases. [36]

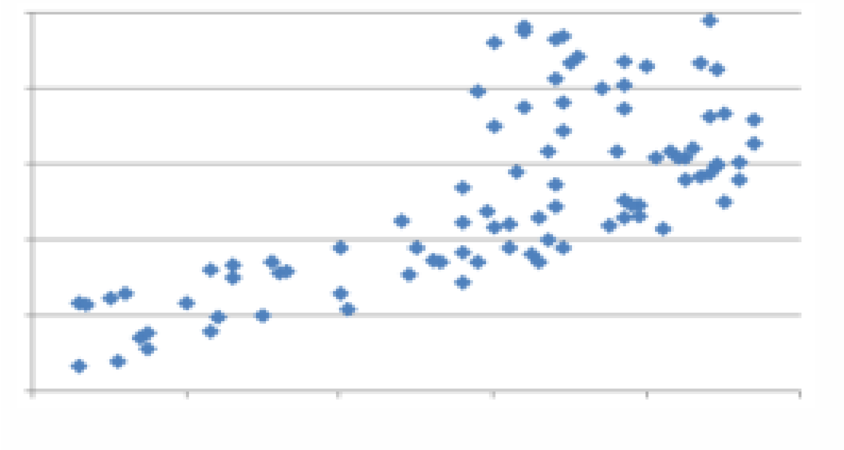

Fig. 1 Example of heteroskedasticity

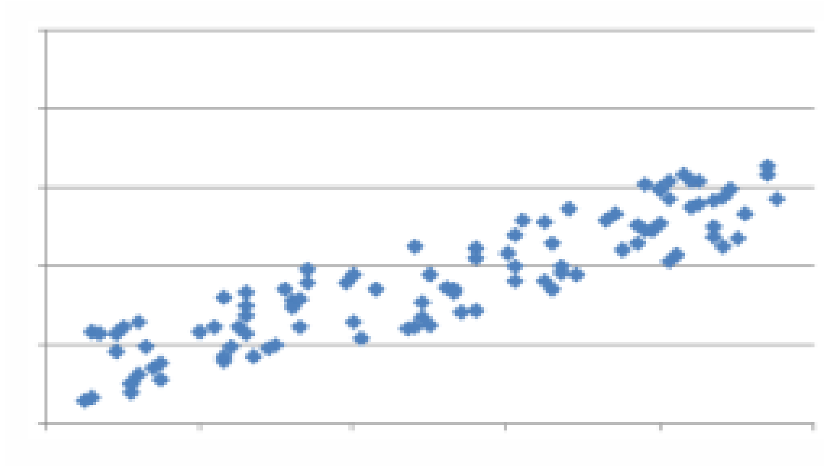

Fig. 2 Example of homoskedasticity

When performing a linear regression, it is assumed that the residual (error) of a regression model is homoskedastic across all values of the predicted value of the dependent variable. [37] When heteroskedasticity is present, the ordinary least squares estimates are unbiased; however, statistical tests of significance may not be accurate. Faulty inferences may be drawn from testing statistical hypothesis when heteroskedasticity is present. [38] When performing a linear regression, the concern about heteroskedasticity is in regards to the error terms, not whether or not there is heteroskedasticity between the independent and dependent variables. [37] Checks for heteroskedasticity can be done visually by graphing the residuals and looking for a significant change in variability.

Additional tests, such as the White, Breusch-Pagan, Goldfeld-Quandt, and Cook-Weisberg tests, can be run in statistical software packages. The White test uses a matrix estimator to compare the "elements of the new estimator to those of the usual covariance estimator" [38: 817]. When heteroskedasticity is not present, the two estimators will be close to equal. If heteroskedasticity is present, the two estimators will diverge. Even if the heteroskedasticity can't be eliminated entirely, correct inferences and confidence intervals can still be obtained by using the White test. [38] The other tests listed above are different in that they require the data to be normally distributed. There are a few ways to correct or reduce heteroskedasticity in a model. The data can be graphed using the log-log plot to see if the variability is stable, which means the data is growing exponentially. Additionally, revisiting the data to consider the original model and whether other variables or possible subgroups are affecting the outcome is an option to adjust for heteroskedasticity. Statistical software packages (such as Stata) have options for estimating robust standard errors, which relax the assumptions that the errors are independent and evenly distributed. Another option for adjusting the model is to use the weighted least squares method, which requires knowing what weights to use to adjust for the variability in the error terms. [39]

Methods of This Study:

Do these ideas and scaling models really hold true for any city, any size, anywhere? Will the scaling laws explored by Bettencourt, et al. apply to cities from the same geographical region with populations below half a million? To explore scaling relations for cities in the United States, data were gathered from nine Metropolitan Statistical Areas (MSAs) and 16 Micropolitan Statistical Areas in the state of Illinois. These statistical areas are not determined by political or geographic boundaries. Instead they are based on unified labor markets, containing an urban core and the surrounding areas where people might be commuting to and from in order to work. According to the U.S. Department of Commerce, Economics and Statistics Administration [40: 5],

> each metro or micro area consists of one or more whole counties and includes the counties containing a core urban area (either a Census Bureau defined urbanized area or urban cluster), as well as any adjacent counties that have a high degree of social and economic integration (as measured by commuting to work) with the urban core. Metro areas contain at least one urbanized area of 50,000 population or more, while micro areas contain at least one urban cluster of less than 50,000, but at least 10,000.

All areas that qualified as a Metro or Micro Statistical Area by the U.S. Census Bureau were included, except for areas that shared a state line, such as the metropolitan area of Chicago, which spills over into Indiana and Wisconsin. The metropolitan and micropolitan population data used (Table 1) was from the 2010 census.

Metropolitan and Micropolitan Statistical Areas of Illinois	Population (2010)
Bloomington	186,133
Carbondale-Marion	126,575
Champaign-Urbana	231,891
Danville	81,625
Decatur	110,768
Kankakee	113,449
Peoria	379,186
Rockford	349,431
Springfield	210,170
Canton	37,069
Centralia	39,437
Charleston-Mattoon	64,921
Dixon	36,031
Effingham	34,242
Freeport	47,711
Galesburg	52,919
Jacksonville	40,902
Lincoln	30,305
Macomb	32,612
Mount Vernon	38,827
Ottawa-Peru	154,908
Pontiac	38,950
Rochelle	53,497
Sterling	58,498
Taylorville	34,800

Table 1 Illinois cities and their populations

Using the U.S. Bureau of Economic Analysis (BEA) and the U. S. Census Bureau, data were gathered on six urban indicators: new patents, total wages, total housing, total employment, gasoline stations, and GDP (Gross Domestic Product). The data are all from the range of 2000 to 2013, with most of the data representing the year 2012. Using Excel, plots of population size versus each urban indictor were generated (Fig. 3-7). All graphs showed a curved function or almost linear function that was best fit with a power law regression, according the Excel regression tools.

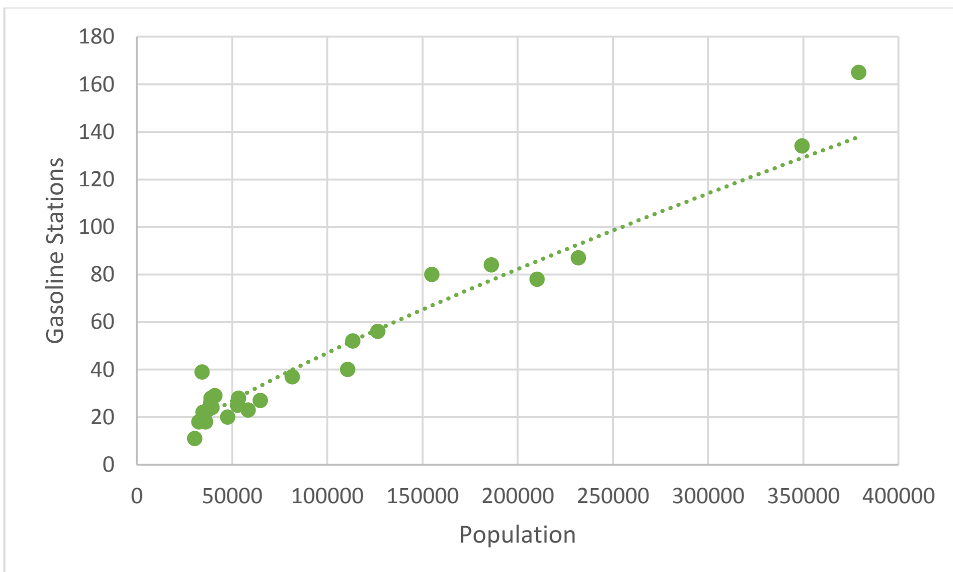

Fig. 3 Number of gasoline stations in 2012 per Metro and Micro SA vs. Population

Sublinear Relationship $y = .004x^{.807}$

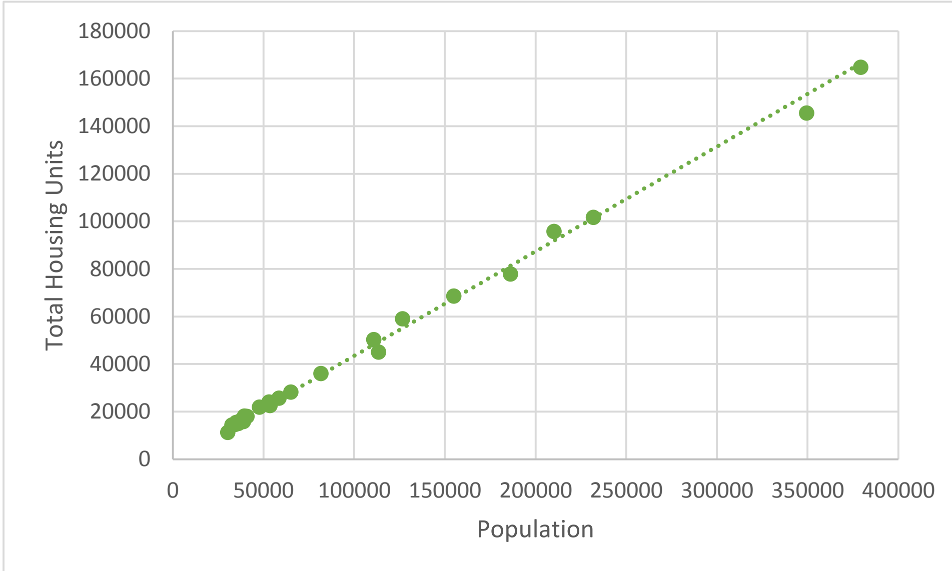

Fig. 4 Number of housing units in 2011-2013 per Metro and MicroSA vs. population

Linear Relationship $y = .3989x^{1.007}$

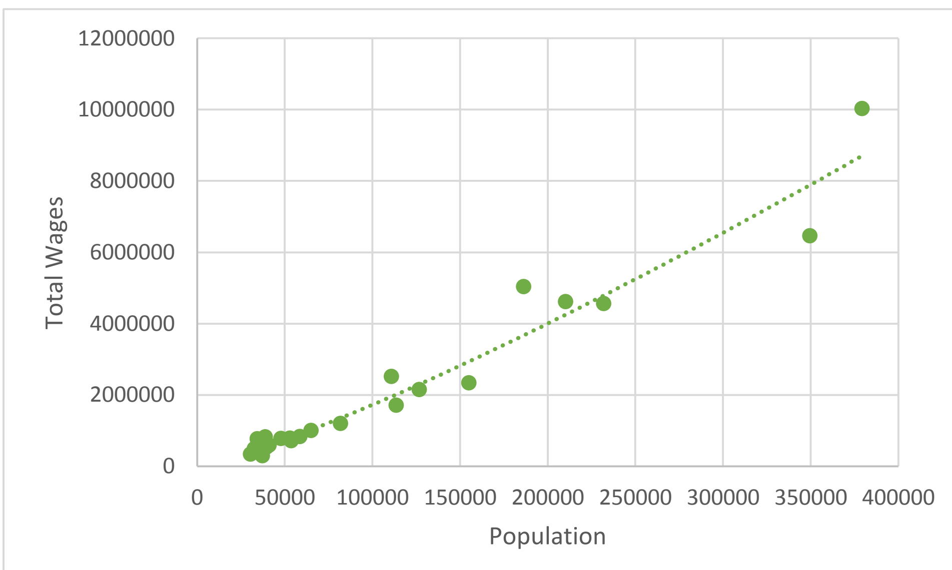

Fig. 5 Total wages earned per Metro and Micro SA in 2012 vs. population

Superlinear Relationship $y = 1.449x^{1.215}$

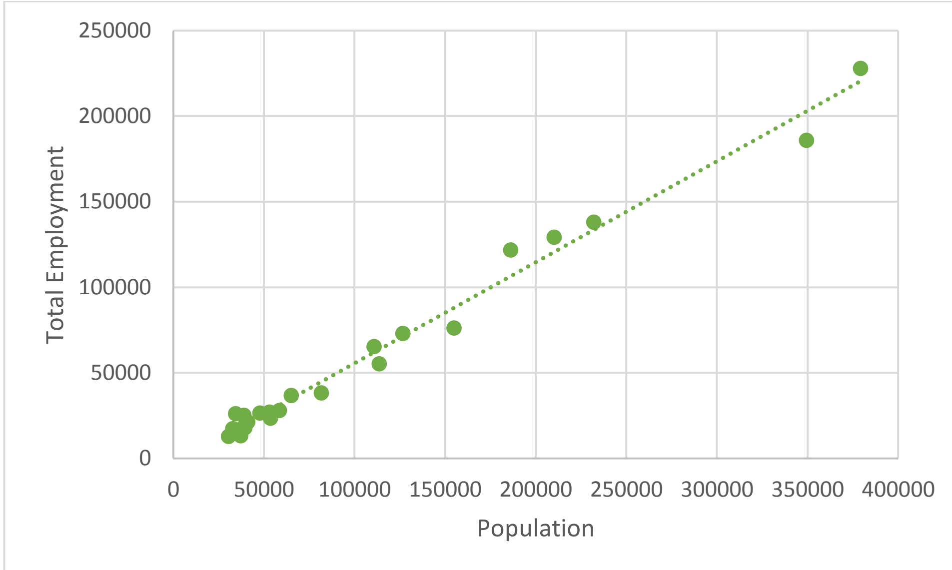

Fig. 6 Total employment per Metro and Micro SA in 2012 vs. population

Superlinear Relationship $y = .225x^{1.076}$

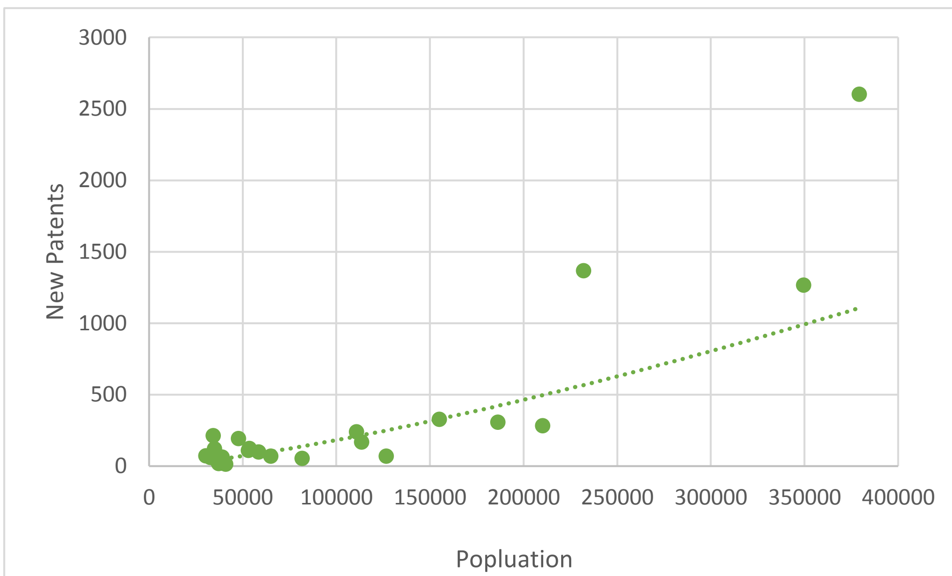

Fig. 7 Number of new patents registered per Metro and Micro SA from 2000-2013 vs. population

Superlinear Relationship $y = .00002925x^{1.358}$

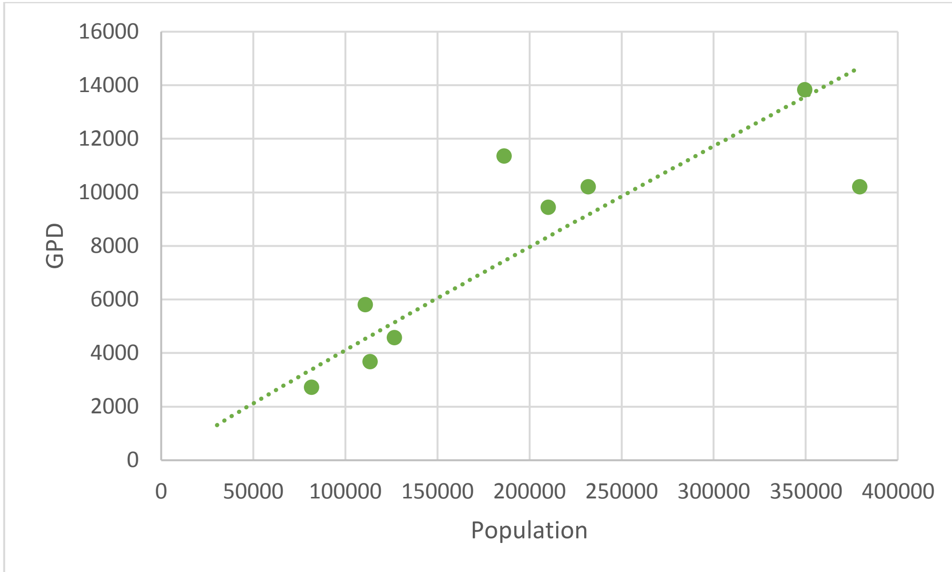

Fig. 8 GDP (in millions) in 2013 per Metro Statistical Area vs. population

Sublinear Relationship $y = 0.0694x^{0.955}$

To verify the power law regressions and compute more statistical values for each urban indicator, the data were transformed into a linear relationship by taking the natural log of the population and the indicators, as seen in Fig. 9-14.

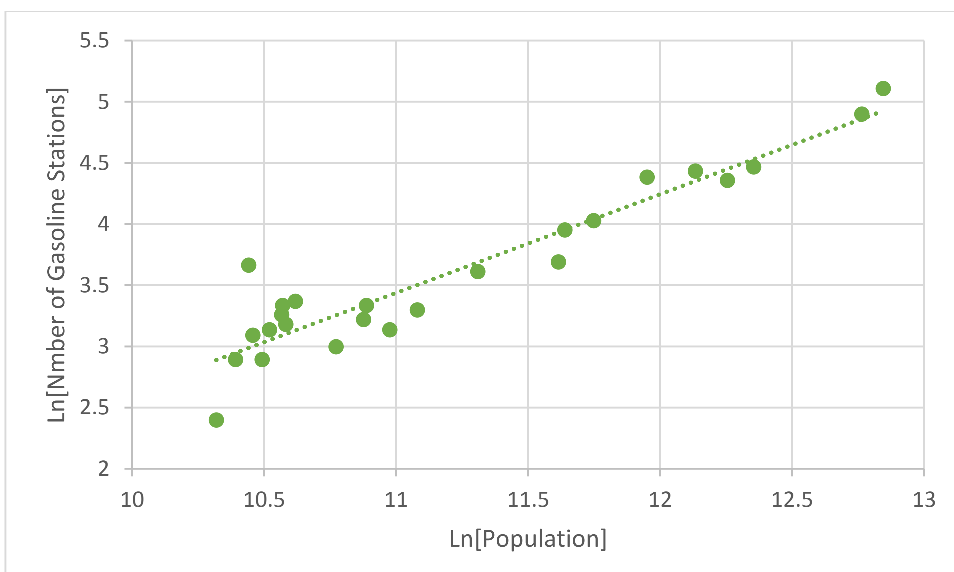

Fig. 9 Number of gasoline stations per Metro and Micro SA in 2012 vs. population

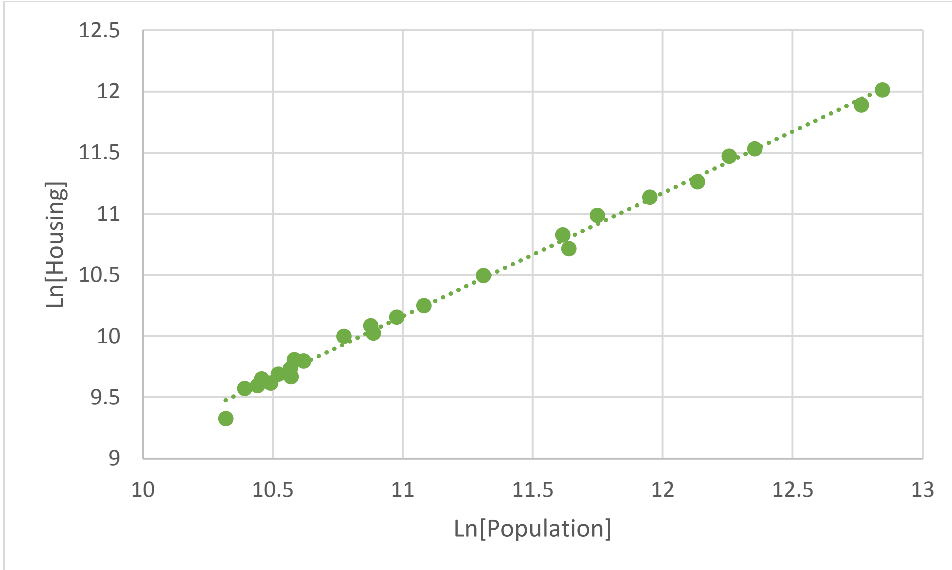

Fig. 10 Total housing units per Metro and Micro SA in 2012 vs. population

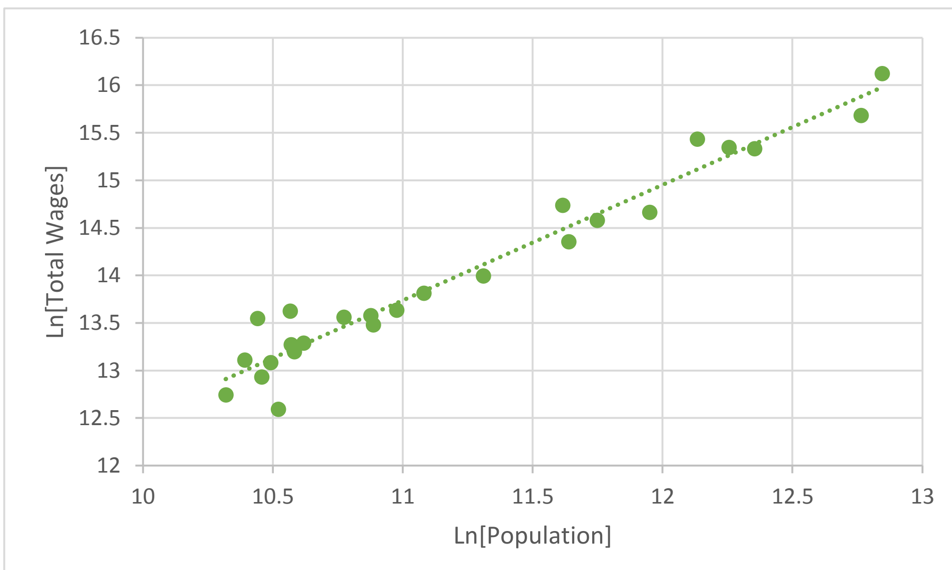

Fig. 11 Total wages per Metro and Micro SA in 2012 vs. population

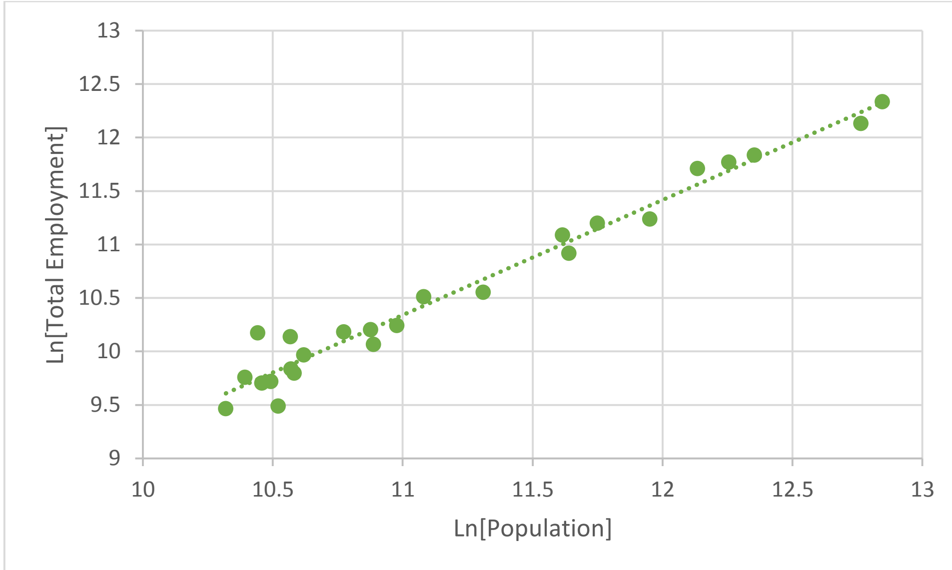

Fig. 12 Total Employment per Metro and Micro SA in 2012 vs. population

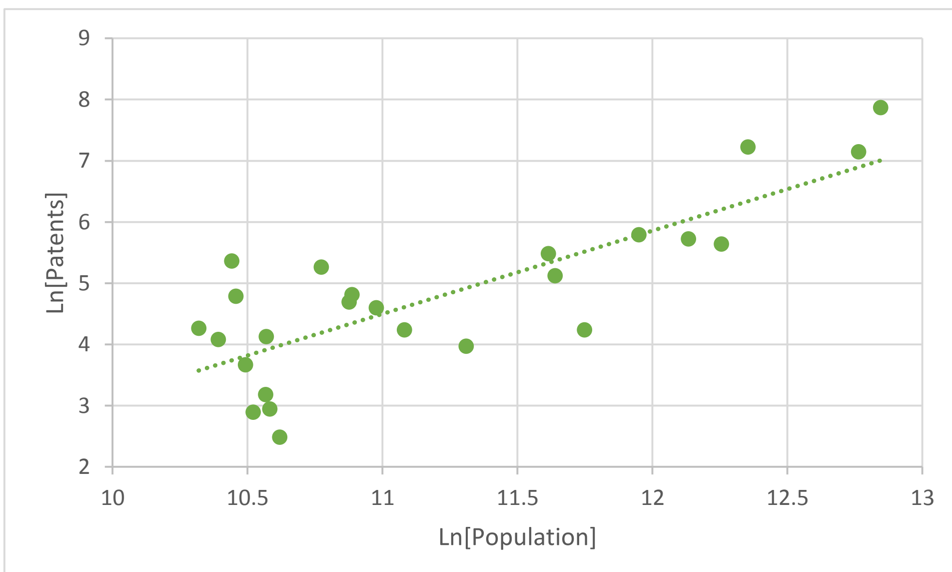

Fig. 13 Number of Patents recorded per Metro and Micro SA from 2000-2013 vs. population

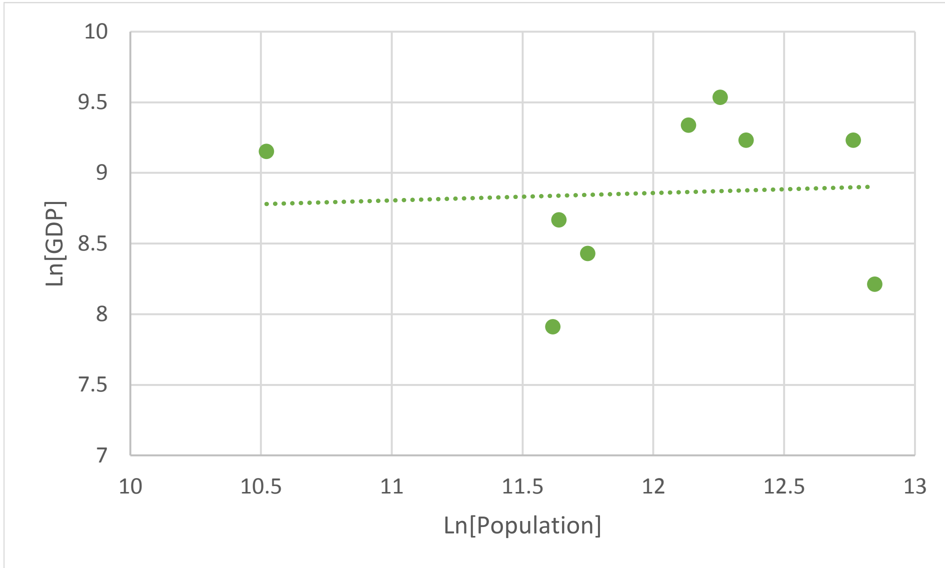

Fig. 14 Gross Domestic Product (GDP) per Metro and Micro SA in 2013 vs. population

Each log transformed set of data and residuals was checked visually for heteroskedasticity. The total housing units, total employment, total wages, and gasoline stations had strong linear relationships with no indications of heteroskedasticity present in the residuals as the population increased. However, the new patent showed signs that the variability of the data increased as population increased. The analysis of the new patents was first done with data for each statistical area from 2011. This data had a few outliers and a glance at the number of patents for each city across the years 2000-2013 showed that each individual city had a great amount of variability from year to year. After looking at using a single year's data and a few different time periods (ex. 2010-2013), the best representation of the patents generated was determined to be the total number of new patents per area over the time period from 2000 to 2013. This lessened the number of outliers in the graph, gave the best confidence interval, and presented more than a yearly snapshot of the data. The GDP was only available for the Metropolitan Statistical Areas in Illinois. This produced a data set with nine observations. The graph shows a great variance in GDP as population increases and it is clear in the log transformed data that the residuals are not consistent. There was not a strong power relationship, thus more data for the urban indicator is necessary to validate any findings.

Results:

Each log transformation was analyzed using an Excel regression statistics add-in that produced regression statistics and ANOVA (analysis of variance divided into various components). After a transformation from the linear log data back to the power law scaling, a common power scaling law was found. Using population, *N(t)*, as the measure of city size at time *t*, the power law scaling creates the equation $U(t) = U_0 N(t)^{\beta}$, where U denotes urban indicators. U_0 is a constant that normalizes the power relationship (normalization constant) and β reflects the dynamics of the urban indictor as city size increase or decreases. Confidence intervals and adjusted R^2 values for the function were found using the regression statistics and ANOVA from the transformed data. Graphs of the residuals did not show signs of heteroskedasticity for any urban indicator.

y	β	95% CI	Adj-R^2	Observations	Year
New Patents	1.36	[.93, 1.79]	0.63	25	2000-2013
Total Wages	1.21	[1.10, 1.33]	0.95	25	2012
Total Employment	1.08	[.99, 1.16]	0.97	25	2012
Total Housing	1.01	[.98, 1.04]	0.99	25	2011-2013
Gasoline Stations	0.81	[.68, .93]	0.88	25	2012

Table 2 Results for power scaling models

In the research by Bettencourt, West, and Lobo [18], any indicators with a β value less than 1.05 was classified as linear and the β values of 1.07 and higher were placed in the superlinear category. The distinction for an approximately linear relationship depends on the context, size of the data, and the impact that β has as the population increases. In this study, $\beta < .95$ will be considered sublinear, $.95 \leq \beta \leq 1.05$ will be categorized as approximately linear, and $\beta > 1.05$ will be labeled as superlinear.

The β values showed that new patents, total wages, and total employment increase super linearly with population size. Increased urban structures and environments may encourage greater creativity, collaboration, and idea generation and in turn, larger wages for residents. In Illinois, the total wages increase at a rate 21% over linear growth and new patents appear to increase at almost 36% over linear growth. Bettencourt and West [18: 7303] also found that "wages, income, growth [sic] domestic product, bank deposits, as well as rates of invention…all scale superlinearly with city size, over different years and nations with exponents that, although differing in detail, are statistically consistent". They argue that socio-economic qualities all increase, on average, by 15% more than the expected linear growth as city size increases [18].

One interesting idea to consider is how these superlinear indicators are related once population size effects are removed [40]. Some trends exist, such that cities that outperform in income often outperform in patents also. Additionally underperformance in both income and generation of new patents is associated with higher rates of violent crime. [40, 41] There are exceptions to these trends, however, in cities that have low income and little crime or rich cities where there are many violent crimes [40]. Previous research has shown that larger cities have higher levels of productivity [2, 42] and the list of superlinear indicators "strongly suggests that there is a universal social dynamic at play that underlies all these phenomena, inextricably linking them in an integrated dynamical network…" [18: 7303]. As referenced in the beginning of the paper, multiple studies on the psychology of cities support the idea that social networks form and define urban centers [2, 8, 17, 18]. In *The Experience of Living in Cities*, Milgram wrote about the changes in individuals that occur when living in a city and how those individual actions translate into cities as a whole [43]. As more research becomes available, possibly stronger links between the superlinear indicators (wages, income, GDP, bank deposits, patents, walking speed, and employment in creative sectors) [18] can be found, along with a discussion of what other urban indicators might scale superlinearly.

In the data for Illinois, total housing is close to a linear relationship. This makes sense that as populations increase, housing is only needed to increase enough to match the population. Bettencourt and West [18] also found that household electrical and water consumption follow a linear pattern with population increases. These indicators seem to represent individual human needs; other urban values that are likely to follow a linear pattern could include food consumed, number of doctor visits, and need for childcare.

In contrast, the number of gas stations per capita in Illinois is sublinear. More condensed urban areas need fewer gasoline stations and each station can service a larger number of people per location. These results are similar to the ones obtained by Bettencourt and West [18], who also found that the length of electrical cables and road surfaces have a sublinear relationship with population. In an interview [19], West explained that “the bigger the city is, the less infrastructure you need per capita. That law seems to be the same in all of the data we can get at”.

Looking at the statistics found, each β scaling value found for the Illinois cities falls in line with the worldwide research from Bettencourt and West [18]. They found the scaling factor β to be 1.27 for new patents, 1.12 for total wages, 1.00 for total housing, and .77 for gasoline stations. All of these scaling values are within .09 of the models for Illinois. This indicates that the cities in Illinois are scaled versions of each other: Springfield is a nonlinearly scaled up version of Rochelle, to a predicable degree. Further research in the field has shown that the deviations from these scaling laws measure how a city over or under performs when compared to the expectations for its size [40, 44]. The notable model in the Illinois data is the new patents, particularly regarding the confidence interval and adjusted-R^2 value. There are two cities (Peoria and Rockford) that have very similar populations; however, Peoria produced 936 new patents in the fourteen year time period, while Rockford only generated 466 new patents. Also, a smaller city (Champaign-Urbana), created 582 new patents in the same time period. The next highest patent data point is 150 new patents. These three cities show variability in the model; perhaps individual factors, such as corporations or universities, impact those cities' patent production. This scaling model is not as strong as the others due to these three high producing cities.

The results from this study of Illinois cities showed that the cities are, in fact, scaled models of each other and that some aspects of urban areas increase in a superlinear fashion, while others increase in a sublinear way. These scaling laws are similar to those found in biology; however, West [19] expressed concern

> that part of what has made life on Earth so unbelievably resilient – able to evolve and survive across billions of years – is the fact that its growth is generally sublinear, with the exponents smaller than 1. Because of that, organisms evolve over generations rather than within their own lifetimes, and such gradual change is incredibly stable.

Currently the human population, urban population, and resources are growing superlinearly [2, 3, 14, 15, 18, 37], which causes concern for sustaining infrastructure, living conditions, and resources into the future. A greater understanding of how cities work, evolve, and grow will help prepare for the current and quickly increasing urbanization of the world.

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