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History of Mathematics from the Islamic World

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By

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Abstract

"The early history of the mind of men with regard to mathematics leads us to point out our own errors; and in this respect it is well to pay attention to the history of mathematics." A De Morgan [17]. Learning the history of mathematics is crucial to fully understanding the world of mathematics today. This paper will explore the history of mathematics from the Islamic world. It will focus on the contributions of well-recognized mathematicians including, Al-Khwarizmi, Al-Khayyam, Uqlidisi, Kushyar ibn Labban, and Abu Kamil. It will also concentrate on the contributions that the Islamic world had on algebra, beginning with Al-Khwarizmi and his contribution to the developmental of algebraic equations, and Khayyam and his contribution to the geometrization of algebra. This paper will also discuss the ways in which the Muslims applied the mathematics they learned into their lives. This paper will provide its readers with a strong foundation on the history of math from the Islamic world which will better enable its readers to fully understand the mathematics we use today.

Prophet Muhammad (peace be upon him) stated, “Seeking knowledge is a duty on every Muslim” (Bukhari) [23]. Thus, there are many Muslim scholars who were keen on doing their part. From the 9th-15th century, Islamic science and mathematics flourished. Throughout history, Muslims from different parts of the world have contributed to the development of mathematics. One way this was done was by translating all sorts of knowledge they believed would be beneficial to society. The two main sources the Muslims translated were the works of the Hindus and the Greeks. Thabit ibn Qurra, a Muslim mathematician, translated the works written by Euclid, Archimedes, Apollonius, Ptolemy, and Eutocius. In Baghdad during 810 A.D, he also founded The House of Wisdom, a school which was dedicated to translating books from Greek to Arabic and also creating commentaries on these books. Thanks to these translations, the knowledge of the ancient Greek texts has survived to this day.

Muslim mathematicians have made significant contributions to different parts of mathematics including algebra, geometry, trigonometry, calculus, arithmetic, and so on. The number system and decimal point we use today comes from the Islamic world. Connected to the decimal system come the fundamental operations: addition, subtraction, multiplication, and division, exponentiation, and extracting the root; although these fundamental operations are possible without the use of the Hindu-Arabic decimal system. They are also responsible for the invention of sine and cosine, the ruler, and the compass. The word algebra comes from “Al-Jabr”, which comes from the book written by Muhammad ibn Musa Khwarizmi, *Hisab al-Jabr wa Muqabala.* Al-Khawarizimi was the first to introduce the concept of zero, also known as “cipher” in the Arabic language. De Vaux, a prominent historian stated the following, “By using ciphers, (Arabic for zero) the Arabs became the founders of the arithmetic of everyday life; they made algebra an exact science. The Arabs kept alive higher intellectual life and the study of science…" [23] The chart below shows the numbers we use today. Below are the Hindu-Arabic numbers, compared to the number written in the Arabic language.

Hindu-Arabic Numbers	Arabic-language Numbers
0	٠
1	١
2	٢
3	٣
4	٤
5	٥
6	٦
7	٧
8	٨
9	٩

We will now look at some prominent mathematicians that have contributed greatly to the development of mathematics.

Al-Khwarizmi on Algebra

Muhammad ibn Musa al-Khwarizmi was born around 780 AD in Baghdad and died around 850 AD. He was a Muslim mathematician and astronomer, who was known for his major contribution on Hindu-Arabic numerals and concepts in algebra, which we will discuss in more detail. Al-Khwarizmi was one of the first to use zero as a place holder in positional base notation. The word algorithm actually derives from his name.

Al-Khwarizmi was most known for his book on elementary algebra, *Al-Kitāb Al-Mukhtaṣar fī Hisāb Al-Jabr Waʾl-muqābala* ("The Compendious Book on Calculation by Completion and Balancing") which is considered one of the first books to be written on algebra. He also wrote a book where he introduces the Hindu-Arabic numerals and their arithmetic. His third major book, *Kitāb ṣūrat al-arḍ* ("The Image of the Earth") presents the coordinates of localities in the known world, including locations in Africa and Asia. Al-Khwarizmi assisted in the construction of a world map, participated in the investigation of determining the circumference of the Earth, and he found volumes of figures such as spheres, cones, and pyramids. He also compiled a set of astronomical tables based on Hindu and Greek sources. Most of Al-Khwarizmi's work was translated into Latin.

Basic Ideas in Al-Khwarizmi's Algebra

According to Al-Khwarizmi, there are three types of quantities: simple numbers (which we would refer to today as natural numbers), such as 1, 18, and 105; root numbers, which he considers an unknown values and calls them "things" (which we would denote today as x); and wealth, which is the square of the root or unknown, also known as *mal*. This is usually denoted as x^2. Also, he states the six basic types of equations as:

1) Roots equal numbers($nx = m$).
2) Wealth equal roots ($x^2 = nx$).
3) Wealth equal numbers ($x^2 = m$).
4) Numbers and wealth equal roots ($m + x^2 = nx$).
5) Numbers equal roots and wealth ($m = nx + x^2$).
6) Wealth equals numbers and roots ($x^2 = m + nx$).

We will now look at an example from Al-Khwarizmi's work.

Example 1: Solve $x^2 + 21 = 10x$

Note: Nowadays, we would simply solve this quadratic equation by using what we call the quadratic formula,$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, or by factoring if the problem is factorable. We can also graph the function and use graphing as a method to solve.

Solution: The first procedure Al-Khwarizmi uses in solving this problem is show in Fig. 1, where he first halves the number of roots, where he receives 5. He then multiplies 5 by itself, where he receives 25. Next, he subtracts 21 from this product, where he receives 4. Further, he takes the square root of 4, where he obtains 2, and subtracts that from 5, where he then receives 3.

$$\frac{10}{2} - \sqrt{\left(\frac{10}{2}\right)^2 - 21} \qquad \text{[Fig. 1]}$$

In his second procedure, he takes the exact same steps as in procedure 1, however, this time instead of taking half the roots and subtracting, he takes half the roots and adds this time. This yields the following expression, as shown in figure 2.

$$5 + \sqrt{5^2 - 21} \qquad \text{[Fig. 2]}$$

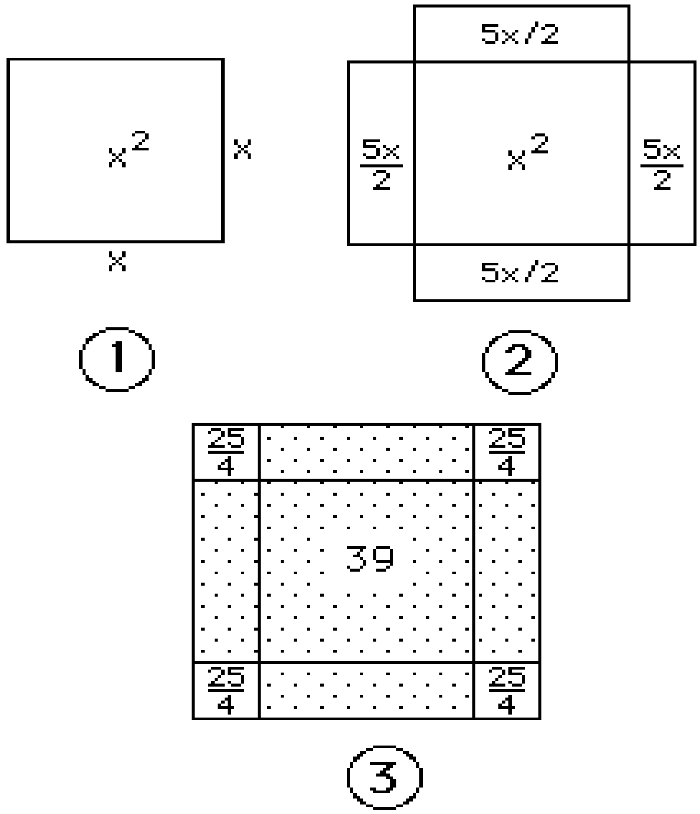
The solution to figure 2 yields 7. In this procedure, he refers to the 10 as "the number of roots", and 21 as the simple number.

Al-Khwarizmi describes the general solution of any quadratic equation of type 4 (as shown above), where *n* represents the number of roots and *m* represents any number as the following…

$$\frac{n}{2} \pm \sqrt{\left(\frac{n}{2}\right)^2 - m} \quad \text{[Fig. 3]}$$

He stated that there were no solutions whenever he received a number less than zero under the square root. Nowadays, we call these numbers imaginary. He also acknowledges that when the number under the square root is equal to zero, then only one solution exists. Also, whenever Al-Khwarizmi had a coefficient in front of px^2, he would divide by p, obtaining $x^2 + \left(\frac{m}{p}\right) = \left(\frac{n}{p}\right) x$. This shows that his coefficients were not restricted to whole numbers only.

We will now turn to another example focusing on the fifth basic types of equation. In this example, we have $39 = x^2 + 10x$, where we have the number equals roots and wealth. Al-Khwarizmi uses an algebraic proof and a geometric proof. We will first look at the algebraic proof which is as follows: The first step is to take half of the roots, 10, which gives us 5. We then multiply it by itself, which is 25. We then add this to 39, where we receive 64. We take the square root of 64, which is 8 and subtract it from it half the roots, 5, which leaves us with 3, our solution.



[Fig. 4]

Next, we will take a look at his geometric proof. In the first step, Al-Khwarizmi starts with a square, where each side length is represented by *x.* Therefore, the area of the square is x^2 (figure 4). Now that we have x^2, we must now add $10x$. We do this by adding four rectangles, each $\frac{10}{4}$ or $\frac{5}{2}$ in length and length x to the square. Here we now have $x^2 + 10x$, which in our example equals to 39 (figure 4). Last, Al-Khwarizmi finds the area of the four little squares, which is $\frac{5}{2} \times \frac{5}{2}$ which gives us $\frac{25}{4}$. Thus, the outside square of figure 4 has an area of $\frac{25}{4} \times 4 + 39$ since the area of the 4 squares are $\frac{25}{4}$ and we have the $x^2 + 10x$ left which we already know is equal to 39. Solving for the area, we receive $25 + 39,$ which equals 64. Therefore, the side length of the square is 8, since the square root of 64 is 8 . The side length is equal to $\frac{5}{2} + x + \frac{5}{2}$.

This can be seen from figure 4 where the two squares have a side length of $\frac{5}{2}$. Therefore, $x + 5 = 8,$ so $x = 3.$ This technique works because once we find the area of the square above, we can use that to determine what the x-value would equal by determining its square root.

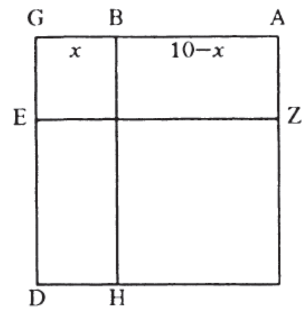
Abu Kamil on Algebra

Abu Kamil Shuja ibn Aslam was born in about 850 AD, most likely in Egypt, and died in 930 AD. He was a Muslim mathematician who was referred to as the "Egyptian Calculator" during the Islamic Golden Age, which was a period that occurred during the middle ages in which much of the historical Arab world experienced a scientific and economic flourishing. It occurred during the 8th century until about the mid 13th century. Abu Kamil is considered to be the first mathematician to use and accept irrational numbers as solutions and as coefficients to equations. Leonardo Bonacci, a twelfth century European mathematician, adopted his mathematical techniques, which allowed Abu Kamil to play an important role in introducing algebra to Europe, even after his death. He worked on and solved non-linear simultaneous equations with three unknown variables. Abu Kamil was one of the first Muslim Mathematicians to work with powers higher than two; the highest power he worked with was the eighth power. He understood that x^5 can be expressed in terms of squares, as x^2x^2x. For x^6, he used cubes and expressed it as x^3x^3.

Abu Kamil wrote many books on mathematics during his lifetime. Some of these books include, but are not limited to the following: *Kitāb fī al-jabr wa al-muqābala (Book of Algebra)*, *Kitāb al-ṭarā'if fi'l-ḥisāb* (*Book of Rare Things in the Art of Calculation), Kitāb al-mukhammas wa'al-mu'ashshar (On the Pentagon and Decagon),* and *Kitāb al-misāḥa wa al-handasa (On Measurement and Geometry*). In his first book, *Book of Algebra,* Abu Kamil discusses and solves problems including, but not limited to, the application of geometry dealing with unknown variables and square roots, quadratic irrationalities, polygons, indeterminate equations, and recreational mathematics. His book, *Book of Rare Things in the Art of Calculation,* provides a number of procedures on finding integral solutions and indeterminate equations. In *On the Pentagon and Decagon,* Abu Kamil calculates the numerical approximation for the side of a regular pentagon in a circle. Lastly, his book *On Measurement and Geometry* contains a set of rules for calculating the volume and surface area of solids. We will now look at some of the examples in his work.

Abu Kamil demonstrates rules and properties of numbers such as $\boldsymbol{ax \times bx = ab \times x^2}$ and $\boldsymbol{a \times (bx) = (ab) \times x}$. He also shows an example of the distributive property where he shows that: $\boldsymbol{(10 - x) \times (10 - x) = 100 + x^2 - 20x}$. Abu Kamil solves this problem algebraically and geometrically, we will look at his geometric proof.

Proof: In figure 5, let line GA be equivalent to 10 in length and GB, x.



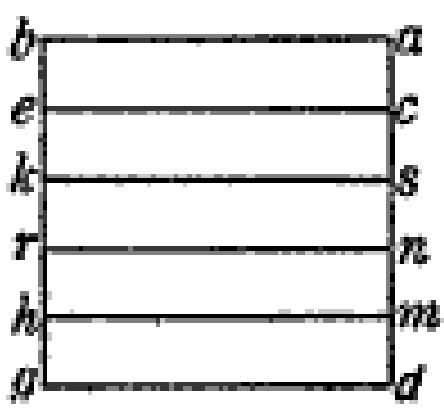
[Fig. 5]

By constructing the square AD on the segment GA, we will get that AB = ED = $\mathbf{10-x}$. Therefore, the square (ZH) = $\mathbf{(10-x)^2}$, and (GZ) = (GH) = $\mathbf{10x}$. Hence, (EH) = (GH) – (EB) = $\mathbf{10x-x^2}$. Therefore, we have that (EH) + (GZ) = $\mathbf{20x-x^2}$ and we know that the large square is 100 so we have the following:

$$\mathbf{(10-x)^2=(ZH)=100-(20x-x^2)=100+x^2-20x}.$$

<u>Abu Kamil's Illustration on Roots</u>

Assume we have the problem: a square is equal to five of its roots, $\mathbf{x^2=5x}$. The root of the square is always equal to the roots to which the square is equal to, in our case, $\mathbf{5x}$. For $\mathbf{x^2}$, we draw a square, *abgd*, and then divide it into 5 equal rectangles, as shown if figure 6.



[Fig. 6]

Take note that lines *be, ek, kr, rh, and hg* are all equivalent and all equal to 1. Therefore, the line *bg* is equivalent to 5. Hence, the area of the square is 25, and from the figure above we can see that the square of 25 is 5. If we multiply the side length *ab* by the side length *be,* that would give us the surface of *abec,* which is the root of the square *abgd.* The surface of the square *abgd* is equivalent to five times the root of itself, or five roots.

Abu Kamil on the Rule of False Position

To solve the following problem, Abu Kamil uses the algebraic device known as "the rule of false position", which is the term used for the method used to evaluate a problem by using "test", or false values for the given variables, and then adjusting them accordingly. The problem below will show us an example of this.

Example 1: Find a quantity that if increased by its seventh part is equal to 19.

Solution: We have the following algebraic equation: $x + \frac{1}{7}x = 19$. Using false position, we plug in 7 for *x* (we use *x* because it is easy to work with since it eliminates our fraction) and obtain the following: $7 + \frac{1}{7} \times 7$ which equals 8, rather than 19. Therefore, we will then divide 19 into 8 and then multiply the result by 7. We will set this as the following proportion: $\frac{19}{8} = \frac{x}{7}$. The reason we do this is because when we plug in 7 the receive 8 as a solution. So the question remains what must we plug in, in order to receive 19. Once we set up this proportion and solve it for *x*, we receive 16.625.

Let's look at another example using false position.

Example 2: Solve the systems of equations: $7y = 13x + 4$ (1)

$4y = 2x + 176$ (2)

Solution: We will use false position and have $y_1 = 40$. Plugging in for equation (1) we receive $7(40) = 13x + 4$. Solving for *x*, we receive that $x_1 = 21\frac{3}{13}$. Then, we plug in x_1 and y_1 into the second equation where we receive $4(40) = 2(21\frac{3}{13}) + 176$, where we get that $160 = 218\frac{6}{13}$. Next, we find the difference of these two numbers: $218\frac{6}{13} - 160 = 58\frac{6}{13} = d_1$.

Using false position again, we will plug in 80 for y_2. Plugging in for equation (1) we receive $7(80) = 13x + 4$. Solving for *x*, we receive that $x_2 = 42\frac{10}{13}$. Then, we plug in x_2 and y_2 into the second equation where we receive $4(80) = 2(42\frac{10}{13}) + 176$, where we get that $320 = 261\frac{7}{13}$. Next, we find the difference of these two numbers: $261\frac{7}{13} - 320 = -58\frac{6}{13} = d_2$. The last step is to solve for *y* by doing the following: We take our y_2 and multiply it by our d_1 and multiply our y_1 and d_2. Then we find the difference between the two. Once we have that, we divide this number by the difference of d_1 and d_2. This can be seen by the following equation:

$$y = \frac{80\left(58\frac{6}{13}\right) + 40(58\frac{6}{13})}{(58\frac{6}{13} + 58\frac{6}{13})} = 60$$

Replacing *y* with 60 in equation (1), we receive that *x* is equal to 32.

Al-Uqlidisi's on Hindu Arithmetic

Abu'l Hassan Ahmad ibn Ibrahim Al-Uqlidisi's was an Arab mathematician who was born around 920 AD in Damascus and died in 980 AD in Damascus. He traveled widely and met and studied from many mathematicians he met throughout his traveling. He was the author of *Kitab al-Fusul fi al-Hisab al-Hindi (The Book of Chapters on Hindu Arithmetic)* and *Kitab al-hajari fi al-hisab(The Book of Records on Arithmetic).* In his work, Uqlidisi focuses on the positional use of Arabic numerals and decimal fractions, where we will look at a couple of his examples below. His treaty on arithmetic is divided into four sections.

In the first part of the treaties, Uqlidisi introduces the Hindu numerals and explains the place value system. He describes addition, multiplication and other arithmetic operations on integers and fractions in decimal notation. In the second part of the treatise he collects arithmetical methods given by earlier mathematicians and converts them in the Indian system. In the third part, Uqlidisi answers questions the reader may have such as "why do it this way?" or "how can I solve this?" and so on. Some of these questions involve understanding the justification in performing several arithmetic steps involved in manipulating problems. Other examples of some of the questions asked are "how do we check what we need to check" or "how do we extract roots of numbers". In the last part, he claims that up to this work, the Indian methods have been used with a blackboard in order to erase and move numbers around as the calculation of the numbers took place. He also showed how to modify these methods when using pen and paper.

Al-Uqlidisi's work is one of the earliest known texts on how to deal with decimal fractions. For example, to halve 19 successively, Al-Uqlidisi wrote the following: 9.5, 4.75, 2.375, 1.1875, and 0.59375. Another example of Uqlidisi is where he increases 135 by its tenth, then the result by its tenth, etc. five times. He first starts by writing $135 \times \left(1+\frac{1}{10}\right)$. Next, changing the mixed number to an improper fraction, he receives $\frac{135\times11}{10}$. He then gets 148.5. Next he gets $148.5\ \times(1+\frac{1}{10})$, which equals to $\frac{148.5\ \times 11}{10}$. He splits this up as $148\times\frac{11}{10}$ and $0.5\times\frac{11}{10}$. He calculates $148\times\frac{11}{10}$, which equals 162.8 and $0.5\times\frac{11}{10}$, which equals 0.55. He adds them to get 163.35, which is his answer.

After studying Uqlidisi's works, Saidan stated, "The most remarkable idea in this work is that of decimal fraction. Al-Uqlidisi uses decimal fractions as such, appreciates the importance of a decimal sign, and suggests a good one". [3]

Al-Uqlidisi's was recorded to discover the multiplication of two mixed numbers. He changed the mixed numbers into improper fractions and multiplied across. In the example below, we will show exactly how Uqlidisi multiplied two mixed numbers.

Example 1: Multiply 7 and a half by 5 and a third. What is shown below shows how Uqlidisi set up such problems.

7	by	5
$\frac{1}{2}$		$\frac{1}{3}$

To solve, we first multiply 7 and 2 and add the one, which becomes $\frac{15}{2}$. We then multiply 5 and 3 and add the one, which becomes $\frac{16}{3}$. Next, we multiply 15 and 16; receiving 240, then we divide by 6, which gives us 40.

Here, Uqlidisi is simply changing a mixed number into an improper fraction, then multiplying the numerators across and the denominators across. We use this exact method today; we only set up the problem a bit differently. We would write this problem as $7\frac{1}{2} \times 5\frac{1}{2}$.

Example 2: Multiply $19 + \frac{1}{3} + \frac{1}{4}$ by $13 + \frac{1}{2} + \frac{1}{5}$

19	
$\frac{1}{3}$	$\frac{1}{4}$

13	
$\frac{1}{2}$	$\frac{1}{5}$

To solve, we first add the fractions $\frac{1}{3}$ with $\frac{1}{4}$ and $\frac{1}{2}$ with $\frac{1}{5}$. To add the fractions, early mathematicians would find would find a new denominator, which was done by finding the product of the given denominators, which in our case is 3 and 4 and 2 and 5. After adding the numerators, the fractions were then reduced to lowest terms. Here, we receive $\frac{7}{12}$ and $\frac{7}{10}$, which we write as…

19
$\frac{7}{12}$

by

13
$\frac{7}{10}$

Now, we can simply solve this problem as we have solved the problem in example 1. The outcome would be 32,195 out of 120.

Another way to solve this problem is to multiply the 19 by the product of 3 and 4, then add the sum of 3 and 4 to that product and write it over the product of 3 and 4. We do the same to the other; we multiply 13 by the product of 5 and 2, then add the sum of 5 and 2 to that product and write it over the product of 5 and 2. The reason this works is because by multiplying the whole number by the product of the denominators, we are simply multiplying by a common denominator. Then the reason why we add the sum of the denominators is because if we multiply the fractions by a common denominator, we end up getting both numbers in the numerator, where we would add them (this only applies when we have a 1 in the numerator).

Kushyar ibn Labban's Principles of Hindu Reckoning

Kushyar ibn Labban was a Persian mathematician, geographer, and astronomer born in Gilan in 971 AD and thought to have died in Baghdad in 1029 AD. His main work seems to have taken place during the 11th century. In one of Labban's most major works, the *Jāmiʿ Zīj* (Universal/Comprehensive astronomical handbook with tables), which was influenced by Ptolemy's *Almagest* and al-Battānī's *Zīj*, contains many tables concerning trigonometry, astronomical functions, star catalogs, and geographical coordinates of cities. It comprises four books: calculations, tables, cosmology, and proofs.

One of his most significant contributions was his work on Hindu reckoning. It is described as follows: “Kushyar ibn Labban's Principles of Hindu reckoning ... is singularly important in the history of mathematics, not only for its mathematical content, but also for its linguistic interest and its relation to earlier and succeeding algorisms. It may be the oldest Arabic mathematical text using Hindu numerals, and ibn Labban's concepts reveal considerable originality...” [14] In the *Principles of Hindu Reckoning*, ibn Labban focuses on decimal numbers and discusses the addition, subtraction, multiplication and division of numbers involving decimals. He also provides different methods on constructing exact square roots, as well as approximate methods to calculate the square roots of non-square numbers. He also does the same for exact cube roots and cube root of a non-square number.

In *Principles of Hindu Reckoning,* Labban focuses on different arithmetic operations of numbers and fraction. We will look at a few of his examples. It is important to take note that many of these problems were done on dust boards, making it easy to erase and replace numbers as shown in the examples below.

Example 1: Add 839 to 5625

We write it as follows…

$$\begin{array}{r} 5625 \\ 839 \end{array}$$

[Fig. 7]

We make sure that all our place values are lined up accordingly.

The first step is to add the highest place value common to both numbers. In this example it would be 56 and the 8, where we receive 64. We replace the 56 with the 64 as shown in figure 8.

6425
839 [Fig. 8]

Next, we add the 3 and the 2, where we receive 5. We replace the 2 with the 5 as shown in figure 9.

6455
839 [Fig. 9]

Last, we add the 9 to the 5, where we receive 14. We add the 1 to the 5 in the tens place of 6455 and replace the 5 in the ones place with the 4 where we receive our final solution. This is shown in figure 10.

6464
839 [Fig. 10]

Example 2: Subtract 839 from 5,625.

We write it as follows…

5625
839 [Fig.11]

The first step is to subtract 8 from 6; however, because this is not possible, instead, we subtract 8 from 56, where we receive 48. Hence, this yields the following figure…

4825
839 [Fig. 12]

Next, we subtract 3 from the 2; however, because this is not possible, instead, we subtract it from the 82, where we receive 79. Hence, this yields the following figure…

4795
839 [Fig. 13]

Now, we subtract the 9 from the 5; however, because this is also not possible, will subtract it from 95 instead, where we receive 86. This will leave us with 4,786, our final solution.

Example 3: Halve 5,625.

This problem will be solved using base 60, just as the Babylonians solved many of their problems. The first step is to halve the 5 in the ones place, where we get $2\frac{1}{2}$. We put the 2 in place of the 5 in the ones place of 5,625 and we place the $\frac{1}{2}$ under. We will write 30 instead of $\frac{1}{2}$ because we are using base 60. This yields to the following figure.

5622
30 [Fig. 14]

Next, we halve the 2 in the tens place, where we receive 1 and replace that 2 with the 1. We also halve the 6, where we receive 3 and replace that 6 with the 3, as shown in figure 15.

5312
30 [Fig. 15]

Last, we halve the 5 in the thousands place. We actually halve 50 and receive 25. We place the 2 in place of the 5 and add the 5 from 25 to the 3. This yields our final solution, shown in figure 16.

2812
30 [Fig. 16]

Example 4: Multiply 325 by 243.

We write this as follows…

325
243 [Fig.17]

The first step is to multiply the 3 of the multiplicand by the 2 of the multiplier which gives us 6. We write this as shown in the following figure.

6 325
243 [Fig. 18]

If the product was other than 6 and contained a number in the tens place value, we would have put the number in the ones place on top of the 2 (same position as it is now) and the number in the tens place to the left of it.

Next, we multiply the 3 of the multiplicand by the 4 of the multiplier which gives us 12. We add the ones from the tens place in 12 to the 6, which gives us 7 and put the 2 to the right of it as shown in figure 19.

72325
243 [Fig. 19]

Now we multiply the 3 of the multiplicand by the 3 of the multiplier to give us 9. We replace the 3 of the multiplicand with this 9 and we shift the multiplier one place to the right, as shown in figure 20.

72925
243 [Fig. 20]

Next, we multiply the 2 of the multiplicand (in the tens place) by the 2 in the multiplier to get 4. We add this to the 2 in the multiplicand and get 6. Then, we multiply the 2 in the multiplicand with the 4 in the multiplier and get 8. We add this to the 9 in the multiplicand. Last, we multiply the 2 in the multiplicand with the 3 in the multiplier, where we get 6. We place this 6 in place of the 2 in the multiplicand. We then shift the numbers in the multiplier one place to the right, as shown in figure 21.

77765
243 [Fig. 21]

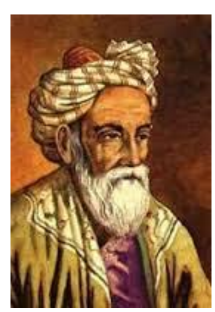
Our final step is to multiply the 5 in the multiplicand by all of the numbers in the multiplier. First, we multiply it by the 2, which gives us 10. We place add the 1 to the 7 in the multiplicand in the thousands place, as shown in figure 6. Then we multiply the 5 by the 4 and receive 20. We add the 2 to the 7 in the multiplicand in the hundreds place, as shown in figure 7. Last we multiply the 5 by the 3 and receive 15. We add the 1 to the 6 in the multiplicand in the tens place and the 5 replaces the 5 in the ones place, where we receive our final solution, as shown in figure 24.

78765
243 [Fig. 22]

78965
243 [Fig. 23]

78975
243 [Fig. 24]

Khayyam



Omar Khayyam was born in Persia in 1048 AD and died in 1131 AD. He was a well-known Persian mathematician, astronomer, philosopher, and poet. Khayyam was well-known for his work in geometry, notably his work on proportions. He completed the algebra treaty, titled “Treatise on Demonstration of Problems on Algebra”. In these treatises he discusses the solution of cubic equations by intersecting conic sections; he intersects a hyperbola with a circle to obtain an answer for a cubic equation. These treatises are considered the first treatment of parallel axioms which is based mostly on intuitive postulates.

Khayyam on the Reform of the Persian Calendar

Khayyam was a part of a panel that introduced several modifications to the Persian calendar; these modifications were accepted as the official calendar of Persia. The Seljuk Sultan Sultan Jalal al-Din Malekshah Saljuqi invited Khayyam to reform the Persian calendar in 1073. Accompanied by other admired scientist, the calendar was completed in 1079, based on Khayyam and other scientists calculations and was known as the Jalili Calendar. The calendar included 2,820 solar years and 1,029,983 days. The Jalili calendar is agreed to be more accurate than the Gregorian calendar because it is based on solar transit, which is the movement of any object passing between the sun and the earth. It also requires an Ephemeris, which is a book that provides the calculated position of celestial objects at intervals throughout a period of time. The Jalili calendar had an error of one day in 3,770 years, whereas the Gregorian calendar has an error of one day for every 3,330 years. Khayyam measured the length of a year as 365.24219858156 days. He rounds his results to the nearest eleventh decimal place; it is clear to see the high level of accuracy Khayyam had.

The Persian calendar is made up of 12 months and they are: Farvardin (31 days), Ordibehesht (31 days), Khordad (31 days), Tir (31 days), Mordad (31 days), Shahrivar (31 days), Mehr (30 days), Aban (30 days), Azar (30 days), Day (30 days), Bahman (30 days), Esfand (29 days in an ordinary year and 30 days in a leap year). The first year begins at vernal equinox, which is when the sun is exactly above the equator and the northern hemisphere starts to tilt towards the sun. If the vernal equinox falls before noon on a particular day, then that day is considered the first day and if it falls after noon, then the next day is considered the first day of the year.

Similarly to the Islamic calendar, years are counted beginning from Muhammad's (peace be upon him) emigration to Medina which took place in AD 622. The Persian calendar also includes leap years, which occurs when there are 366 days between two Persian New Year's days. Because the Persian calendar is based on the vernal equinox, there remain constraints on adjusting the beginning of the calendar to the beginning of the day (midnight). Therefore, the Persian calendar runs short of the tropical year by about 5h, 48m, 45.2s each year. Further, the length of a year shortens by 0.00000615th of a day each century. To make up for these losses leaps years are included mostly every 4 years. Four-year leap years add one-fourth of a day, or 0.25, to each year in the period. However, this is more than what is lost and therefore, there is overcompensation. To overcome this, after every 6 to 7 four-year leap years, there is a five-year leap year, which means the nest leap year occurs after 4 normal years instead of 3.

Application of Mathematics

The Muslims applied the knowledge they gained in mathematics throughout their daily lives. Next, we will look at a few different ways math was used to help people with the calculation of inheritance, Zakat (charity), and with creating art.

Inheritance: The Prophet Muhammad, (peace be upon him), said, “Learn the laws of inheritance and teach them to people, for that is half of knowledge”. [23] In Islam, when a person dies, there are specific requirements on the laws of inheritance. The arithmetic of fractions can be used to solve the calculation of the legal shares of a person who dies and leaves no legacy of the natural heir. We will look at two examples from Al-Khwarizmi’s work to illustrate the arithmetic.

Example 1: “A women dies, leaving her husband, a son, and three daughters, and the object is to calculate the fraction of her estate that each heir will receive.” [9]

Solution: The Islamic law states that, in this case, the husband receives $\frac{1}{4}$ of the estate and that the son receives double the amount the daughter receives. (It should be noted that the son or husband is responsible for the financial well being of their sister or wife, hence.). After the husband takes his share, the remainder of the estate, $\frac{3}{4}$ is then divided into five parts: two for the son and three for the daughters. The least common multiple of five and four is twenty; therefore the estate should be divided into twenty equal parts. Of these, the husband gets five, the son receives six, and each daughter receives three.

Example 2: “A women dies, leaving her husband, son, and three daughters, but she also bequeaths to a stranger $\frac{1}{8}+\frac{1}{7}$ of her estate. Calculate the shares of each.”[9] (As a side note, “the law on legacies states that a legacy cannot exceed one-third of the estate unless the natural heirs agree to it.")

Solution: Since $\frac{1}{8}+\frac{1}{7}\leq\frac{1}{3}$, no complications occur and we can move forward with the calculation. The least common denominator of the legal shares is 20. After the stranger's legacy is paid, which is calculated by adding $\frac{1}{8}+\frac{1}{7}$, this gives us $\frac{15}{56}$, we have $\frac{41}{56}$ remaining. The ratio of the strangers share to the total share of the family is 15: 41. Now we will multiply both numbers by 20, the least common denominator, to compute of the shares of the inheritors. We have $20\times(15+41)=20\times 56=1120$. The stranger receives $20\times 15=300$ and the family receives $20\times 41=820$. The husband receives one-fourth of 820, which is 205; the son receives six-twentieths, which is 246; and each daughter receives the remaining, which would yield 123 for each.

Conclusion

Muslim mathematicians have contributed a great deal of knowledge to the development of mathematics. They have expanded on the mathematical work of other great scholars and have also developed their own mathematical work and ideas. Without their dedication, we may not know some of the information we use to this day.

From Al-Khwarizmi, we are able to learn how he solves different types of quadratic equations, algebraically and geometrically. From Abu Kamil, we learn about how he uses false position to solve equations, as well as using the distribution property by looking at his geometric proof. We also have Uqlidisi, where we learn how he multiplied mixed numbers. From looking at Kushyar's work, we are able to see how the fundamental operations (adding, subtracting, multiplying and dividing) were computed. Lastly, we have Khayyam and his significant contribution to the Persian calendar.

In conclusion, it is clear to see what a great contribution these mathematicians had in the development of mathematics. From there work, we are able to gain an insight on how they solved mathematical problems.

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